gAGE/UPC research group of Astronomy and Geomatics
Barcelona**TECH**, Spain

Lecture 11

Differential positioning with code

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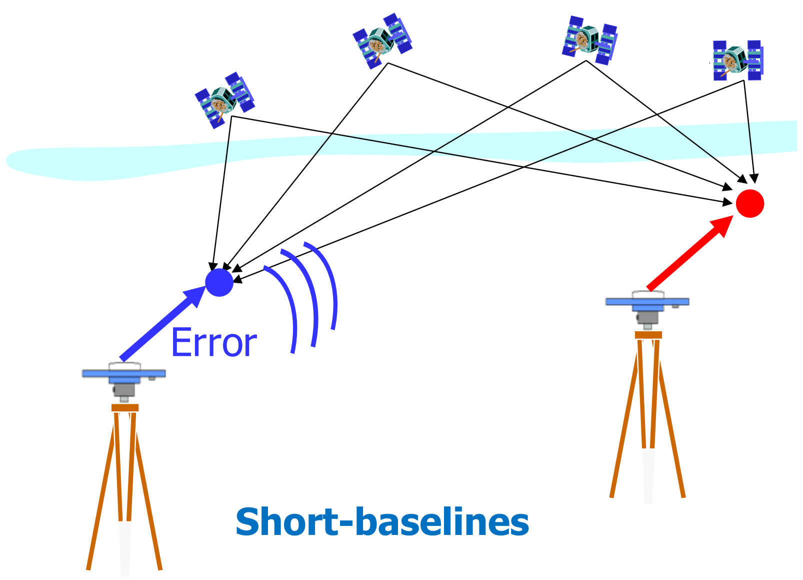
Linear model for DGNSS: Single Differences

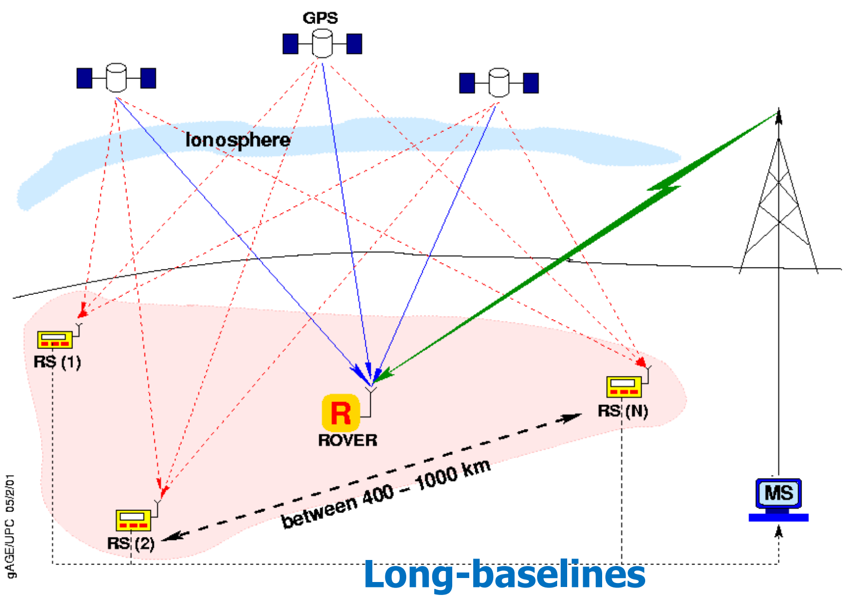
1. Linear model
2. Geographic decorrelation of ephemeris errors
3. Error mitigation and `short´ baseline concept
4. Differential code based positioning

Error mitigation: DGNSS residual error

Errors are similar for users separated tens, even hundred of kilometres, and these errors vary 'slowly' with time. That is, the errors are correlated on space and time.

The spatial decorrelation depends on the error component (e.g. Clocks not decorrelate, ionosphere ~100km...). Thence, long baselines need a reference stations network.





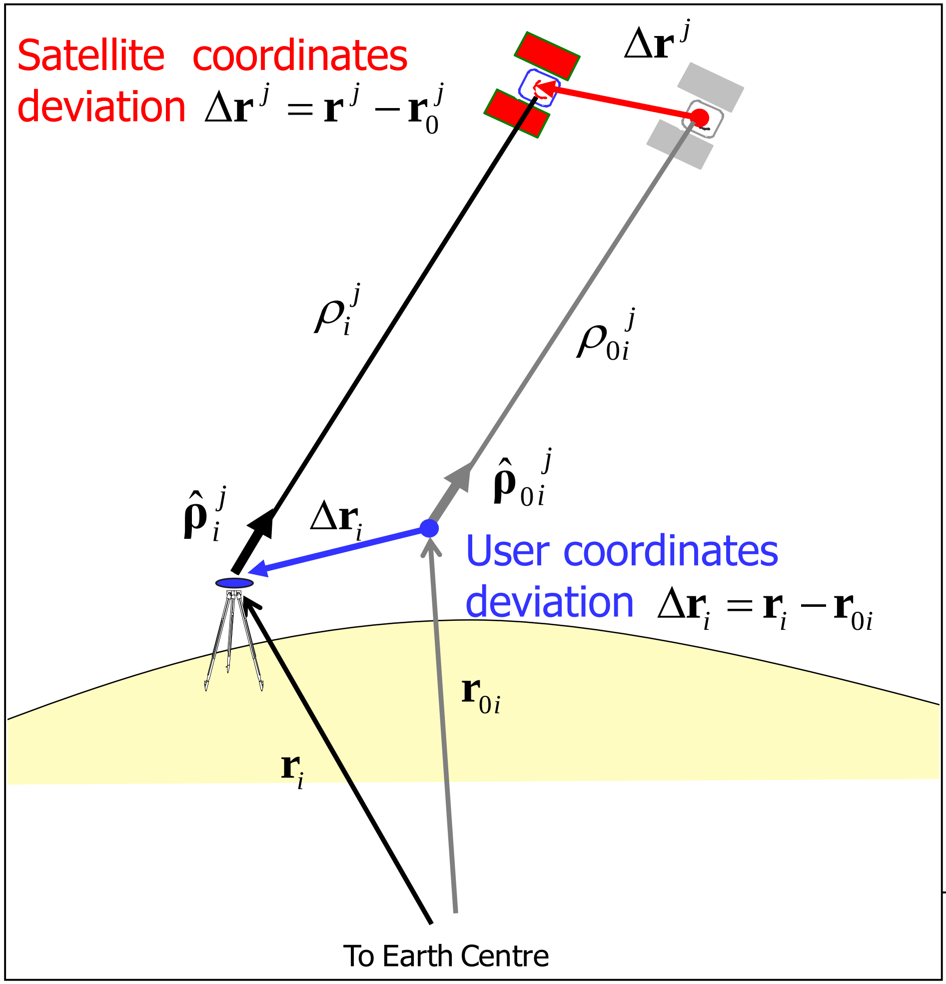
Linear model for Differential Positioning

Code and carrier measurements

$$P_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j + I_i^j + K_i + K^j + \mathcal{M}_i^j + \nu_{P\,i}^{\;j}$$

$$L_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j - I_i^j + \lambda\,\omega_i^j + \lambda\,N_i^j + b_i + b^j + m_i^j + \nu_{L\,i}^{\;j}$$

Noise terms



When approximate values of both receiver and satellite APC positions are taken, a linearization around them yields:

$$\rho_i^j = \rho_{0i}^j - \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}_i + \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}^j$$

$$\hat{\boldsymbol{\rho}}_{0i}^j = \frac{\mathbf{r}_0^j - \mathbf{r}_{oi}}{\left\|\mathbf{r}_0^j - \mathbf{r}_{oi}\right\|}$$

$\Delta\mathbf{r}_i$: Receiver coordinates error

$\Delta\mathbf{r}^j$: Satellite coordinates error

Linear model for Differential Positioning

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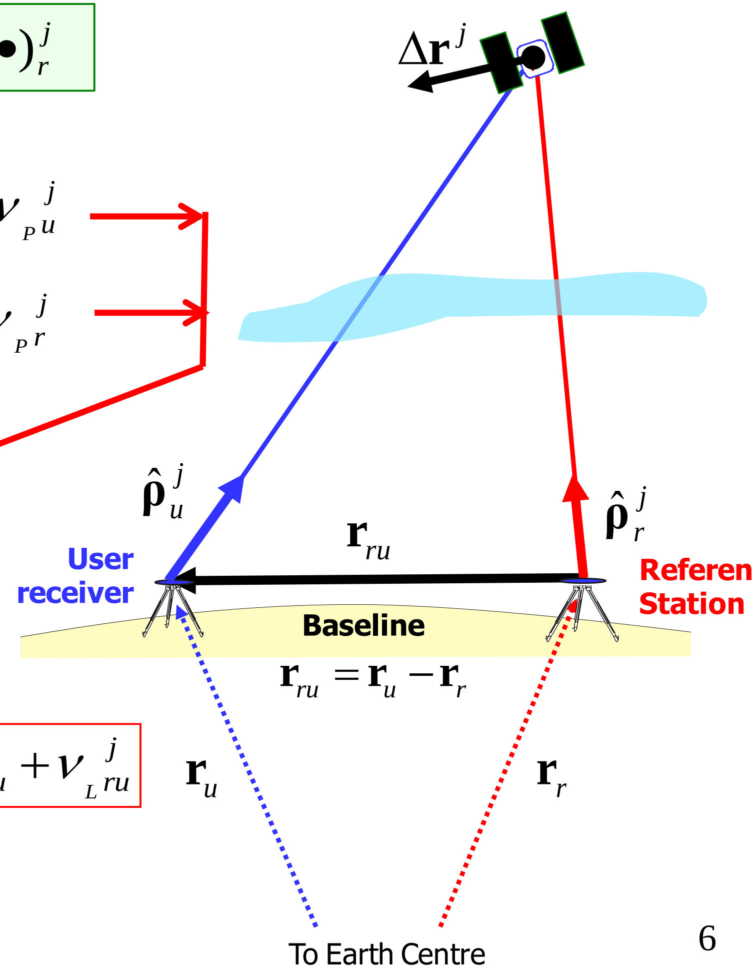
$$L_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j - I_i^j + \lambda\,\omega_i^j + \lambda\,N_i^j + b_i + b^j + m_i^j + \nu_{L\,i}^{\ j}$$

Single difference $(\bullet)_{ru}^j \equiv \Delta(\bullet)_{ru}^j = (\bullet)_u^j - (\bullet)_r^j$

$$P_u^j = \rho_u^j + c\left(\delta t_u - \delta t^j\right) + T_u^j + I_u^j + K_u + K^j + \nu_{P\,u}^{\ j}$$

$$P_r^j = \rho_r^j + c\left(\delta t_r - \delta t^j\right) + T_r^j + I_r^j + K_r + K^j + \nu_{P\,r}^{\ j}$$

$$P_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \nu_{P\,ru}^{\ j}$$



The same for the carrier :

$$L_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda\,\omega_{ru}^j + \lambda\,N_{ru}^j + b_{ru} + \nu_{L\,ru}^{\ j}$$

Linear model for Differential Positioning

Code and carrier measurements

$$P_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j + I_i^j + K_i + K^j + \mathcal{M}_i^j + \nu_{P\,i}^{\ j}$$

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Single difference $(\bullet)_{ru}^j \equiv \Delta(\bullet)_{ru}^j = (\bullet)_u^j - (\bullet)_r^j$

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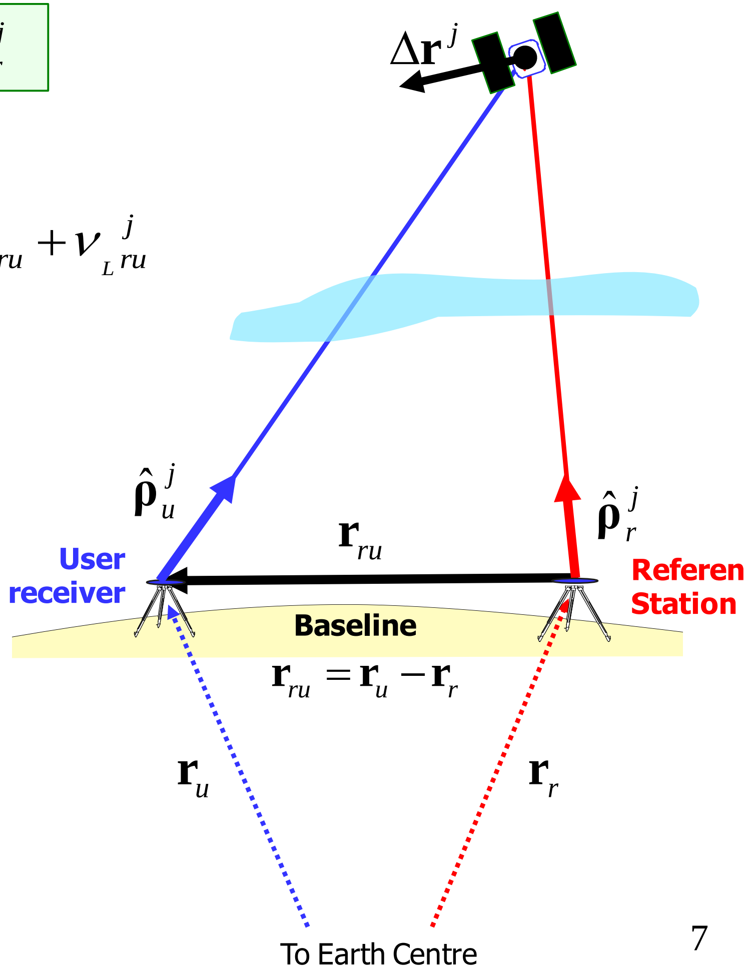
$$L_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda\,\omega_{ru}^j + \lambda\,N_{ru}^j + b_{ru} + \nu_{L\,ru}^{\ j}$$

Single difference cancels:
- Satellite clock (δt^j)
- Satellite code instrumental delays (K^j)
- Satellite carrier instrumental delays (b^j)

Single differences mitigate/remove errors due
- Satellite Ephemeris $(\Delta \mathbf{r}^j)$
- Ionosphere (I_i^j)
- Troposphere (T_i^j)
- Wind-up (ω_i^j)

The residual errors will depend upon the baseline length.



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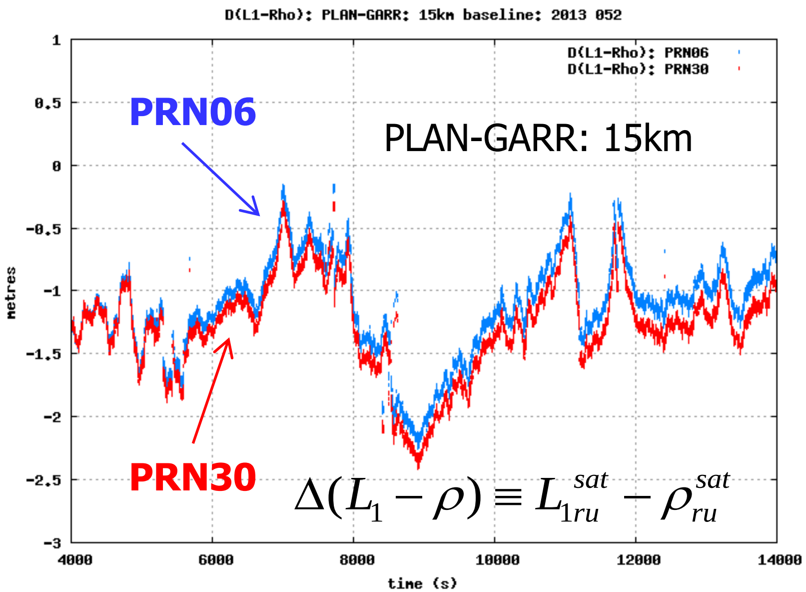
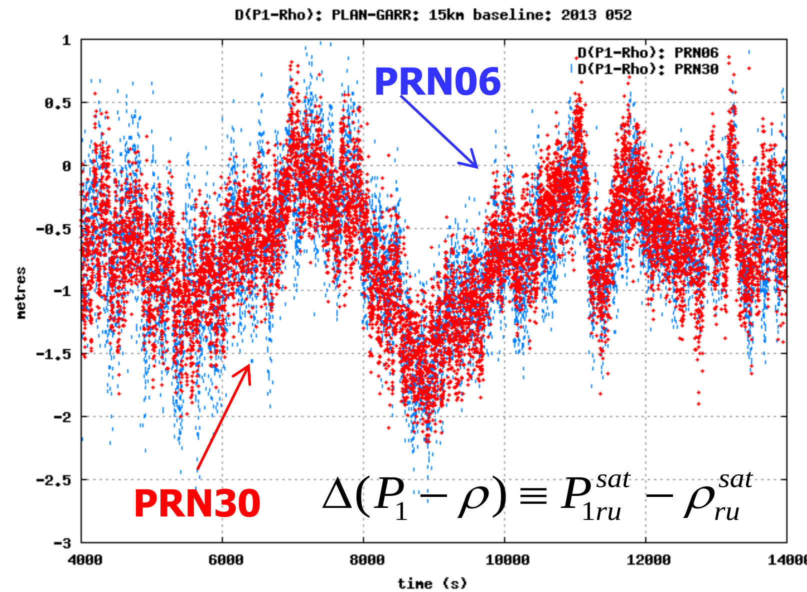
Single-Difference of measurements (corrected by geometric range!!)





Dif. Wind-up: Very small

$$\Delta(L_1-\rho)\equiv L_{ru}^{j}-\rho_{ru}^{j}=c\,\delta t_{ru}+T_{ru}^{j}-I_{ru}^{j}+\lambda\,\omega_{ru}^{j}+\lambda\,N_{ru}^{j}+b_{ru}+\nu_{L\,ru}^{\;j}$$

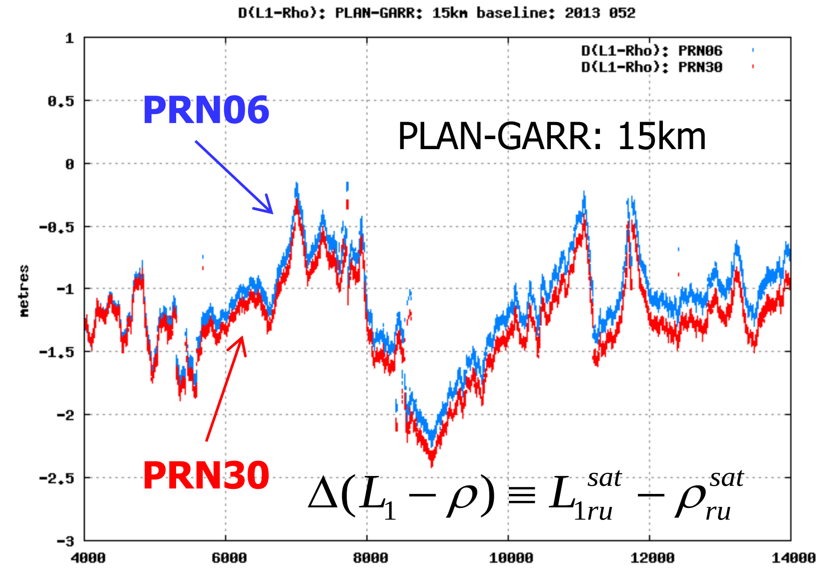
$$\Delta(P_1-\rho)\equiv P_{ru}^{j}-\rho_{ru}^{j}=c\,\delta t_{ru}+T_{ru}^{j}+I_{ru}^{j}+K_{ru}+\nu_{P\,ru}^{\;j}$$

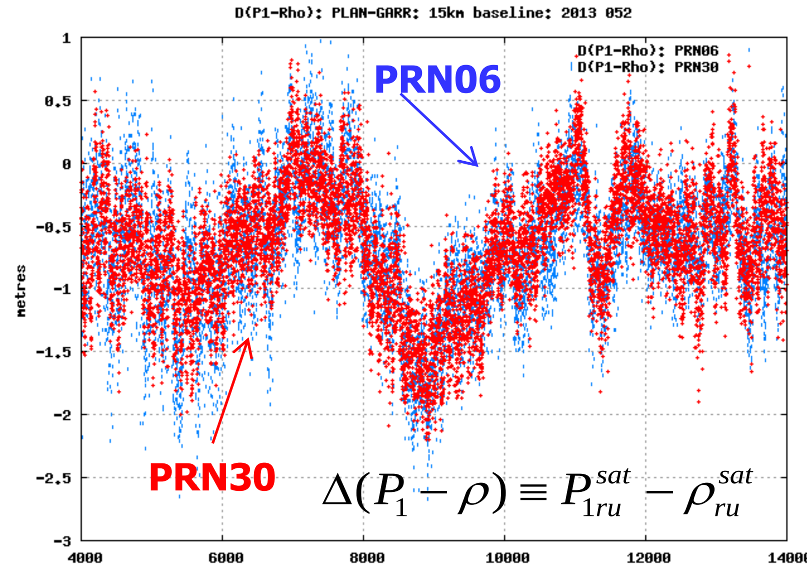
Dif. Instrumental delays and carrier ambiguities: constant

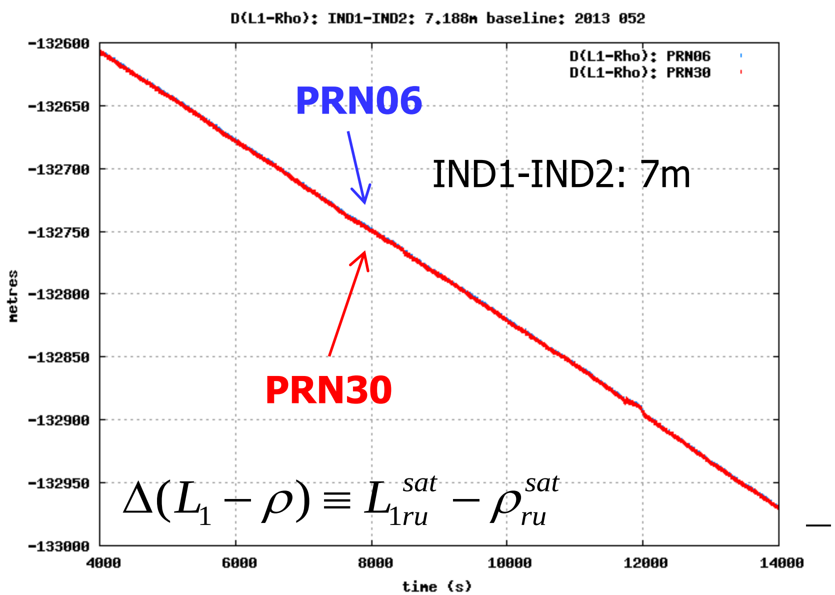
Dif. Receiver clock: Main variations Common for all satellites

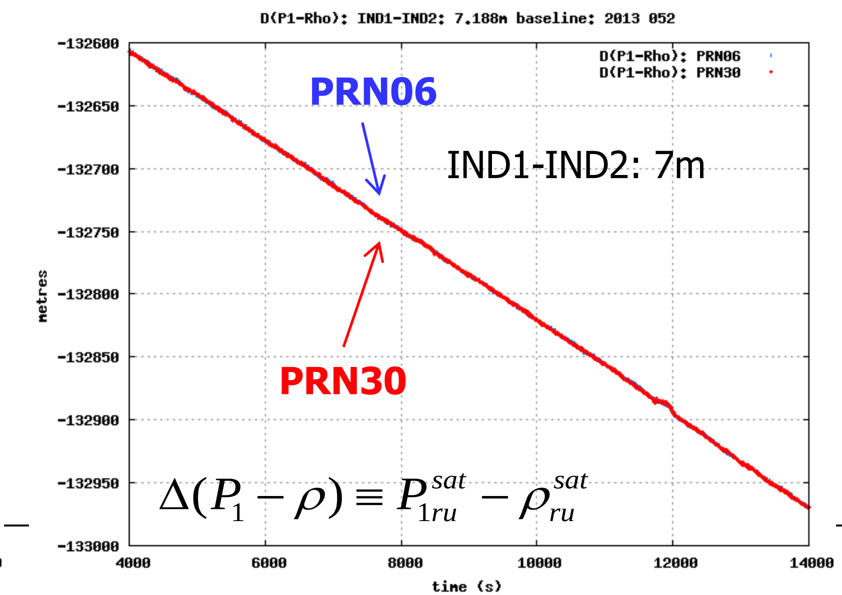
Dif. Tropo. and Iono. : Small variations

Single-Difference of measurements (corrected by geometric range!!)









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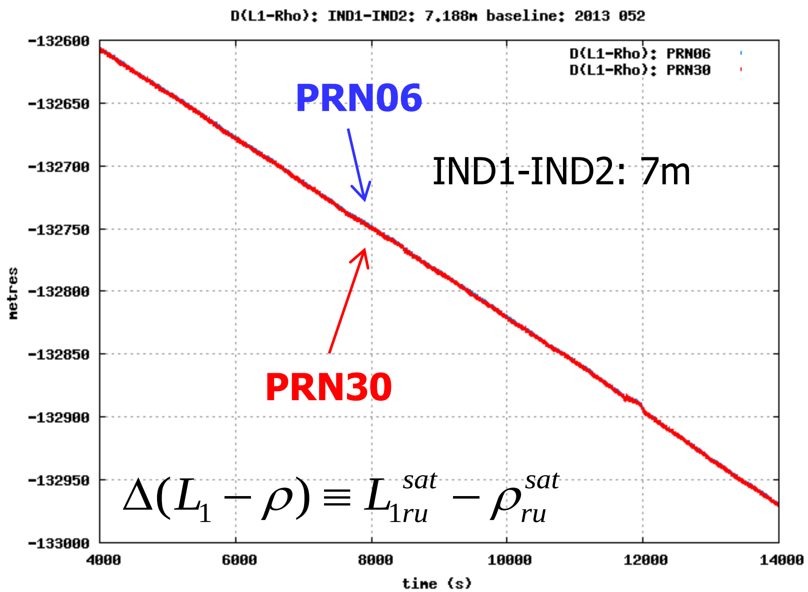
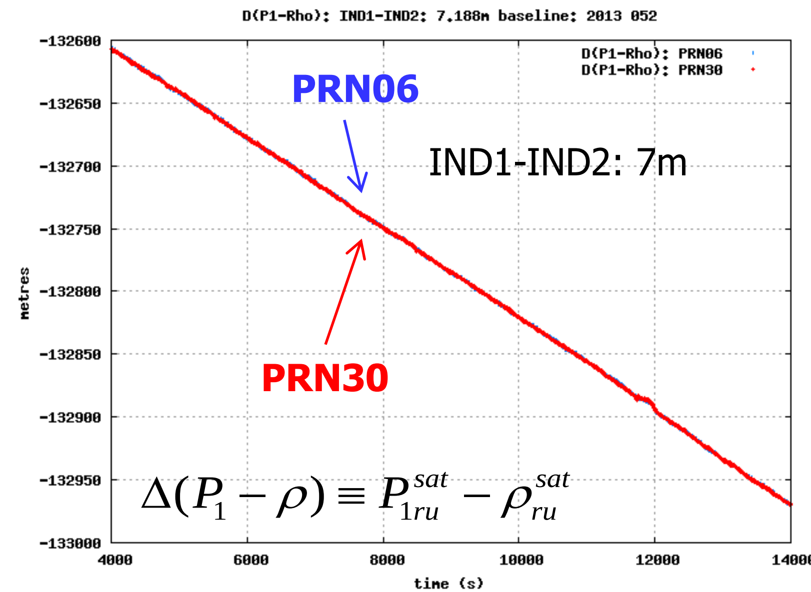
Single-Difference of measurements (corrected by geometric range!!)





Dif. Wind-up: Very small

$$\Delta(L_1-\rho)\equiv L_{ru}^{j}-\rho_{ru}^{j}=c\,\delta t_{ru}+T_{ru}^{j}-I_{ru}^{j}+\lambda\,\omega_{ru}^{j}+\lambda\,N_{ru}^{j}+b_{ru}+\nu_{L\,ru}^{\;j}$$

$$\Delta(P_1-\rho)\equiv P_{ru}^{j}-\rho_{ru}^{j}=c\,\delta t_{ru}+T_{ru}^{j}+I_{ru}^{j}+K_{ru}+\nu_{P\,ru}^{\;j}$$

Dif. Receiver clock: Main variations Common for all satellites

Dif. Tropo. and Iono. : Small variations

Dif. Instrumental delays and carrier ambiguities: constant

Linear model for Differential Positioning

Code and carrier measurements

$$P_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j + I_i^j + K_i + K^j + \nu_{P\,i}^{\;j}$$

$$L_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j - I_i^j + \lambda\,\omega_i^j + \lambda\,N_i^j + b_i + b^j + \nu_{L\,i}^{\;j}$$

where: $\rho_i^j = \rho_{0i}^j - \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}_i + \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}^j$

$$\Delta\mathbf{r}^j \equiv \boldsymbol{\varepsilon}_{eph}^j$$

Single difference

$$(\bullet)_{ru}^j \equiv \Delta(\bullet)_{ru}^j = (\bullet)_u^j - (\bullet)_r^j$$

$$P_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \varepsilon_{ru}^j$$

$$L_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda\,\omega_{ru}^j + \lambda\,N_{ru}^j + b_{ru} + \nu_{L\,ru}^{\;j}$$

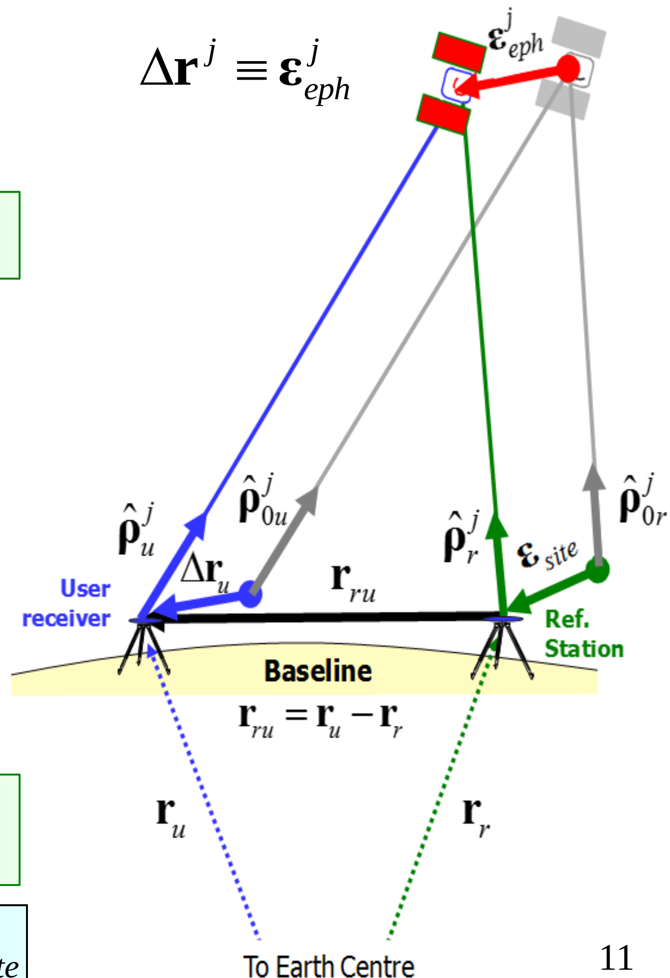
where: $\rho_{ru}^{\,j} = \rho_u^{\,j} - \rho_r^{\,j}$

With some algebraic manipulation, it follows:

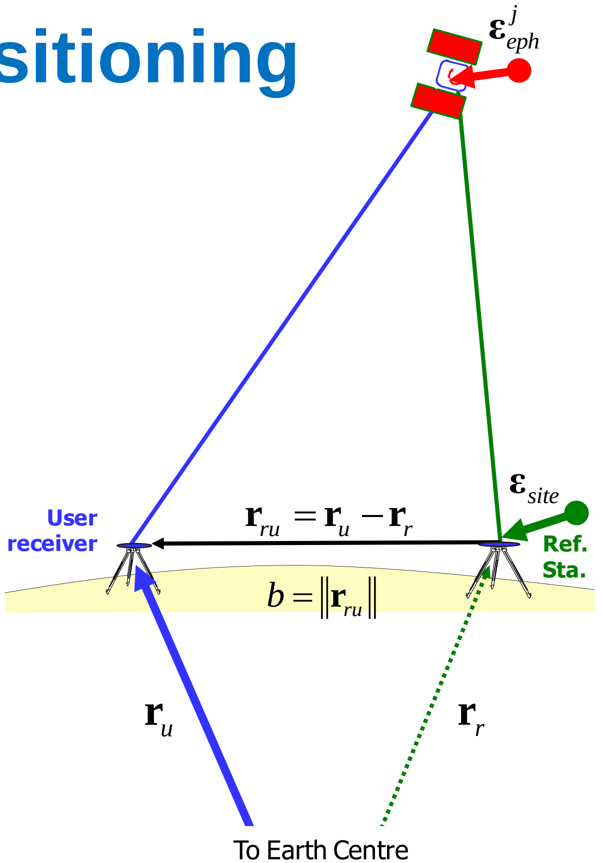
$$\rho_{ru}^j = \rho_{0ru}^j - \hat{\boldsymbol{\rho}}_{0u}^j \cdot \Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{site} + \hat{\boldsymbol{\rho}}_{0ru}^j \cdot \boldsymbol{\varepsilon}_{eph}^j$$

being: $\rho_{0ru}^j \equiv \rho_{0u}^j - \rho_{0r}^j$; $\Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_u - \boldsymbol{\varepsilon}_{site}$

Finally: $\mathbf{r}_u = \mathbf{r}_{0u} + \Delta\mathbf{r}_{ru} + \boldsymbol{\varepsilon}_{site}$



Linear model for Differential Positioning



and where the $\Delta\mathbf{r}_{ru}$ estimate will be affected in turn by the ephemeris and sitting site errors as:

$$\rho_{ru}^{j} = \rho_{0ru}^{j} - \hat{\boldsymbol{\rho}}_{0u}^{j} \cdot \Delta\mathbf{r}_{ru} - \boxed{\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{site}} + \boxed{\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j}}$$

Range error due to **reference station** coordinates uncertainty

Range error due to **Sat. coordinates** uncertainty

Thence, taking into account the relationships:

$$\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{site} \le \frac{b}{\rho_u^j}\|\boldsymbol{\varepsilon}_{site}\| \quad ; \quad \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j} \le \frac{b}{\rho_u^j}\|\boldsymbol{\varepsilon}_{eph}^{j}\|$$

and being $\rho_u^j \approx 20\,000\,\mathrm{km}$ it follows that for a baseline $b = 20\,\mathrm{km}$

$$\frac{b}{\rho_u^j} \approx \frac{1}{1000} \Rightarrow$$

The effect of 5 metres error in orbits or in site coordinates is less than 5 mm in range for the estimation of $\Delta\mathbf{r}_{ru}$.

But, the user position estimate will be shifted by the error in the site coordinates $\mathbf{r}_u = \mathbf{r}_{0u} + \Delta\mathbf{r}_{ru} + \boldsymbol{\varepsilon}_{site}$

Linear model for Differential Positioning

Exercise:

Demonstrate the following relationship:

$$\hat{\boldsymbol{\rho}}_{0ru}^{\,j} \cdot \boldsymbol{\varepsilon}_{site} \le \frac{b}{\rho} \left\| \boldsymbol{\varepsilon}_{site} \right\|$$

Hint:

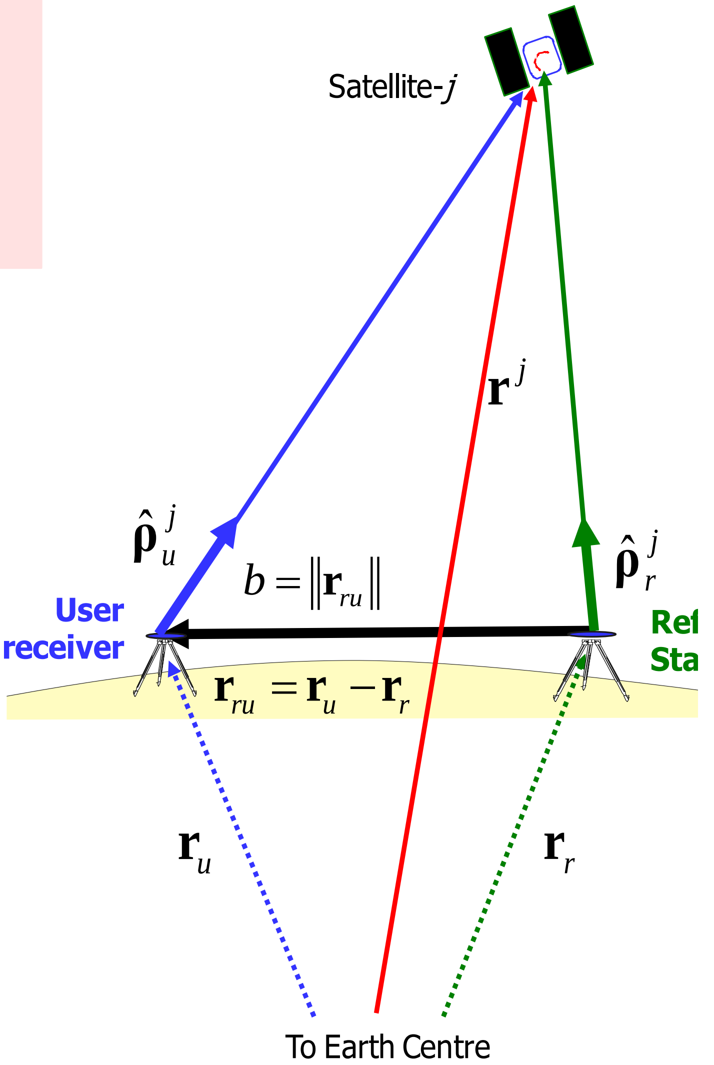
$$\hat{\boldsymbol{\rho}}_{0ru}^{\,j} \cdot \boldsymbol{\varepsilon}_{site} = \left(\hat{\boldsymbol{\rho}}_{0u}^{\,j} - \hat{\boldsymbol{\rho}}_{0r}^{\,j} \right) \cdot \boldsymbol{\varepsilon}_{site}$$

$$\approx \left(\frac{\boldsymbol{\rho}_{0u}^{\,j} - \boldsymbol{\rho}_{0r}^{\,j}}{\rho} \right) \cdot \boldsymbol{\varepsilon}_{site} \le \frac{b}{\rho} \left\| \boldsymbol{\varepsilon}_{site} \right\|$$

Note: the following approaches have been taken:

$$\rho_{0r}^{j} \approx \rho_{0u}^{j} \approx \rho \quad \Rightarrow \begin{cases} \hat{\boldsymbol{\rho}}_{0r}^{\,j} = \dfrac{\boldsymbol{\rho}_{0r}^{\,j}}{\rho_{0r}^{j}} \approx \dfrac{\boldsymbol{\rho}_{0r}^{\,j}}{\rho^{j}} \\ \hat{\boldsymbol{\rho}}_{0u}^{\,j} = \dfrac{\boldsymbol{\rho}_{0u}^{\,j}}{\rho_{0u}^{j}} \approx \dfrac{\boldsymbol{\rho}_{0u}^{\,j}}{\rho^{j}} \end{cases}$$

$$b = \left\| \mathbf{r}_{ru} \right\| \approx \left\| \mathbf{r}_{0ru} \right\| \qquad \mathbf{u} \cdot \mathbf{v} \le \left\| \mathbf{u} \right\| \left\| \mathbf{v} \right\|$$



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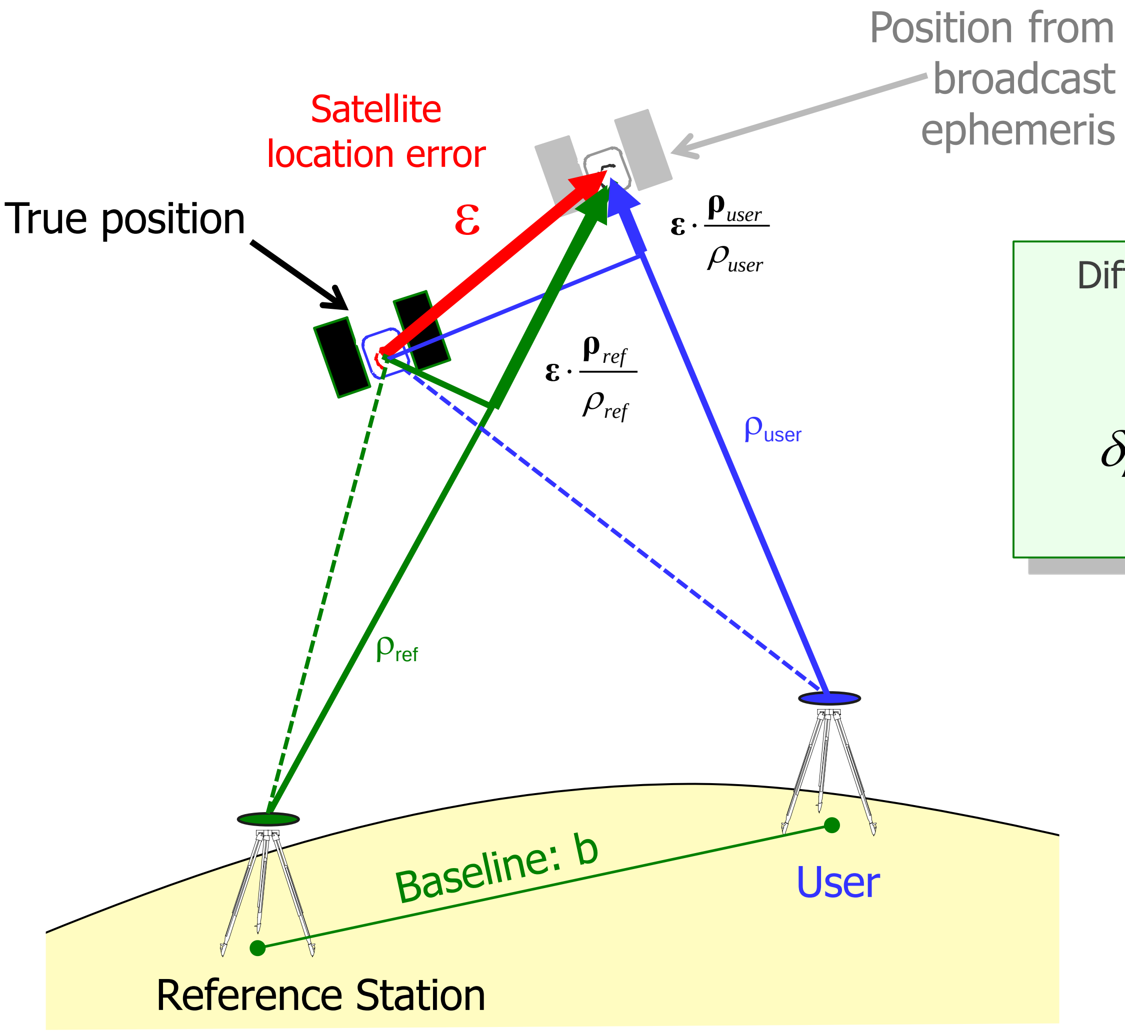
Geographic decorrelation of ephemeris errors



Differential range error due to satellite obit error

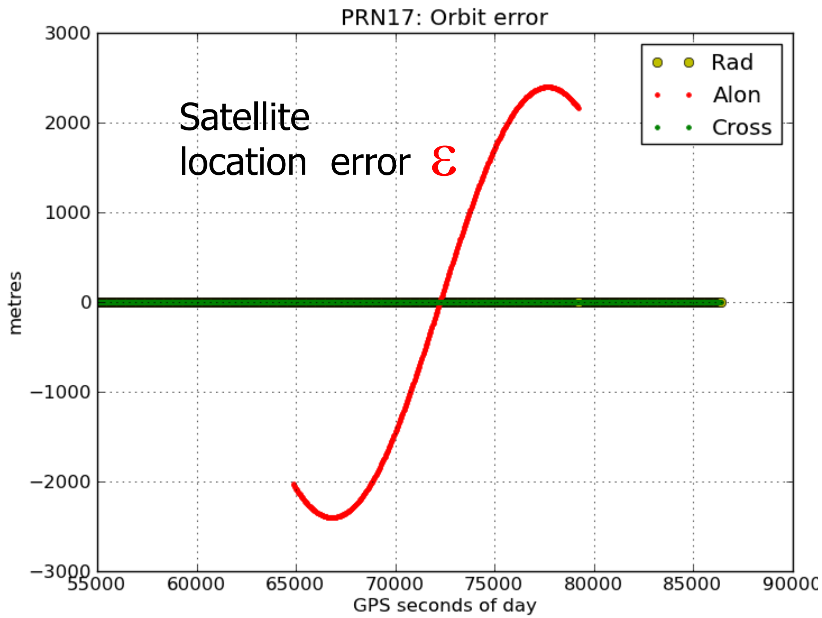
$$\delta\rho = \boldsymbol{\varepsilon} \cdot \frac{\boldsymbol{\rho}_{user}}{\rho_{user}} - \boldsymbol{\varepsilon} \cdot \frac{\boldsymbol{\rho}_{ref}}{\rho_{ref}}$$

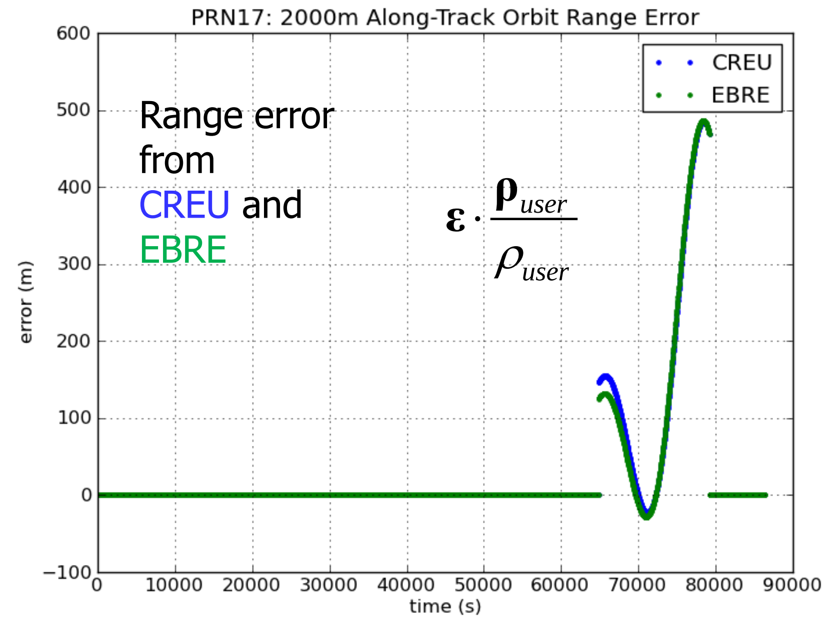
A conservative bound:

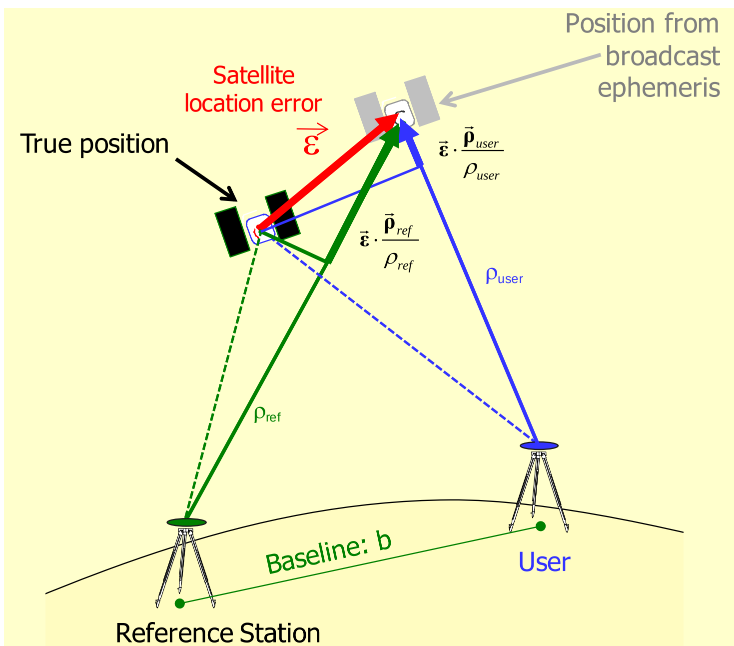
$$\delta\rho < \frac{b}{\rho}\varepsilon$$

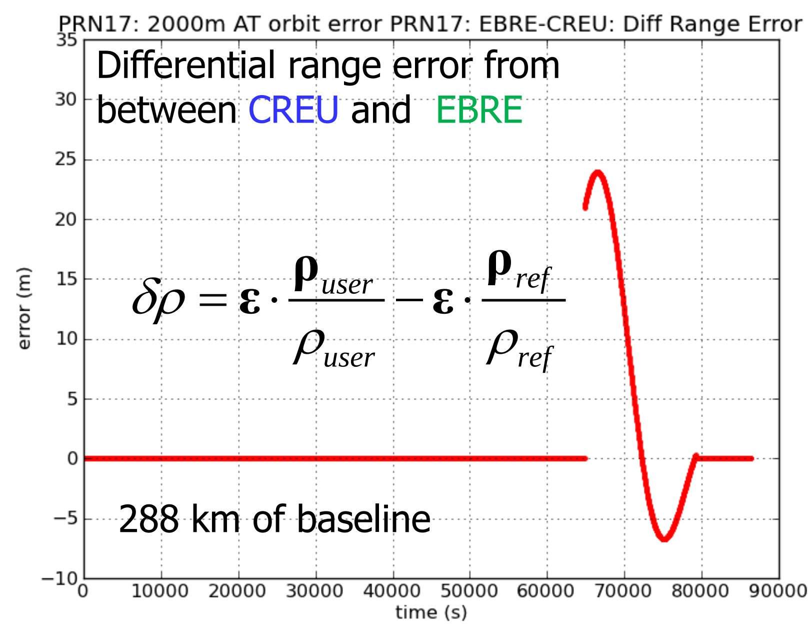
with a baseline $b = 20km$

$$\delta\rho < \frac{20}{20000}\varepsilon = \frac{1}{1000}\varepsilon$$


PRN17: Orbit error
Satellite
location error ε
Rad
Alon
Cross
metres
GPS seconds of day
3000
2000
1000
0
−1000
−2000
−3000
55000
60000
65000
70000
75000
80000
85000
90000


PRN17: 2000m Along-Track Orbit Range Error
Range error
from
CREU and
EBRE
$\boldsymbol{\varepsilon}\cdot\frac{\boldsymbol{\rho}_{user}}{\rho_{user}}$
CREU
EBRE
error (m)
time (s)
600
500
400
300
200
100
0
−100
0
10000
20000
30000
40000
50000
60000
70000
80000
90000


Position from
broadcast
ephemeris
Satellite
location error
$\vec{\varepsilon}$
True position
$\vec{\varepsilon}\cdot\frac{\vec{\rho}_{user}}{\rho_{user}}$
$\vec{\varepsilon}\cdot\frac{\vec{\rho}_{ref}}{\rho_{ref}}$
ρ_{user}
ρ_{ref}
Baseline: b
User
Reference Station


PRN17: 2000m AT orbit error PRN17: EBRE-CREU: Diff Range Error
Differential range error from
between CREU and EBRE
$\delta\rho=\boldsymbol{\varepsilon}\cdot\frac{\boldsymbol{\rho}_{user}}{\rho_{user}}-\boldsymbol{\varepsilon}\cdot\frac{\boldsymbol{\rho}_{ref}}{\rho_{ref}}$
288 km of baseline
error (m)
time (s)
35
30
25
20
15
10
5
0
−5
−10
0
10000
20000
30000
40000
50000
60000
70000
80000
90000

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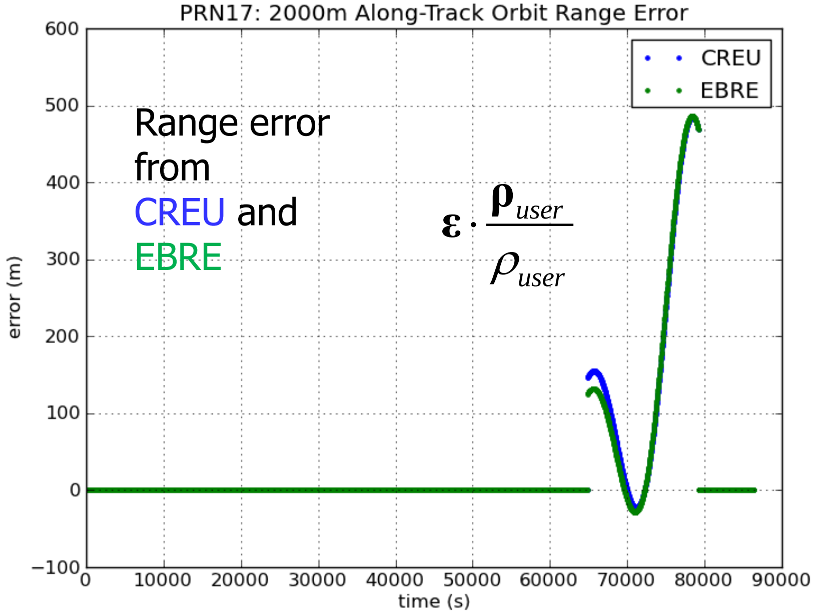
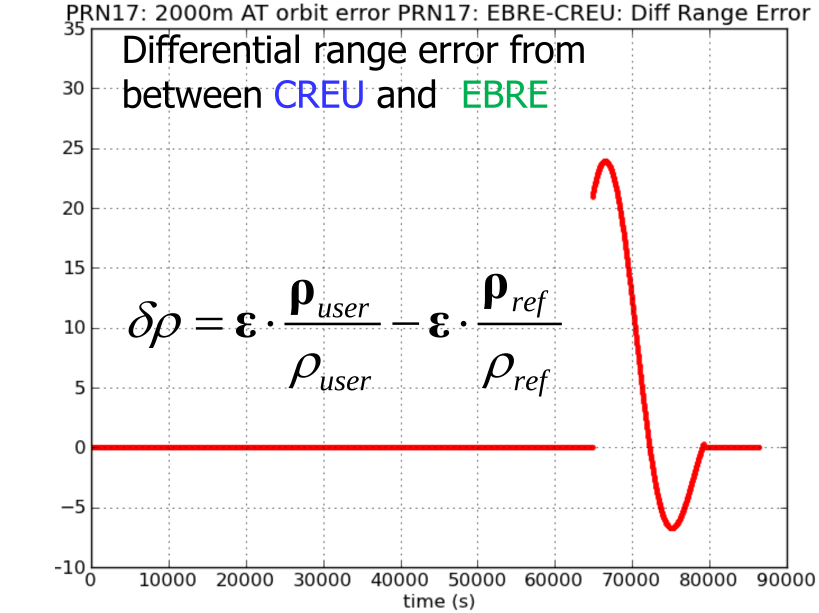
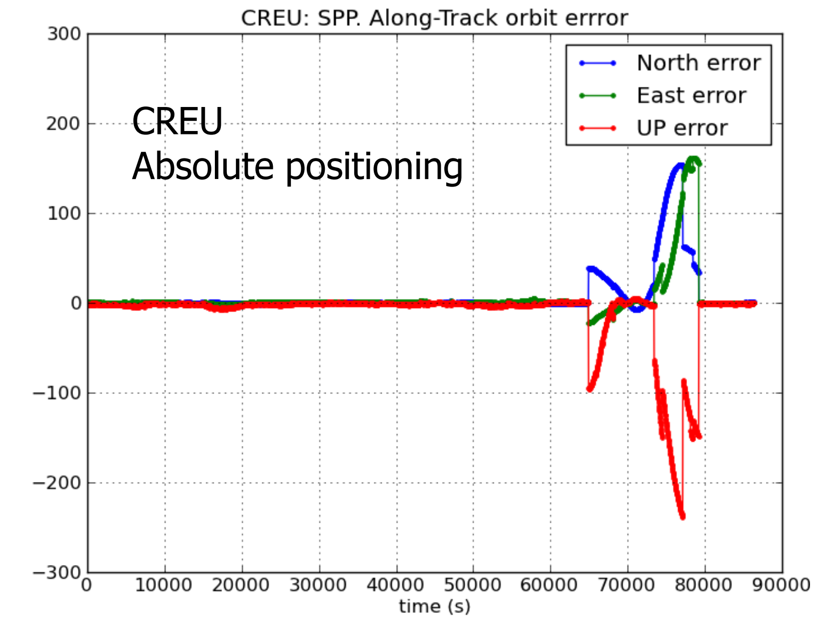
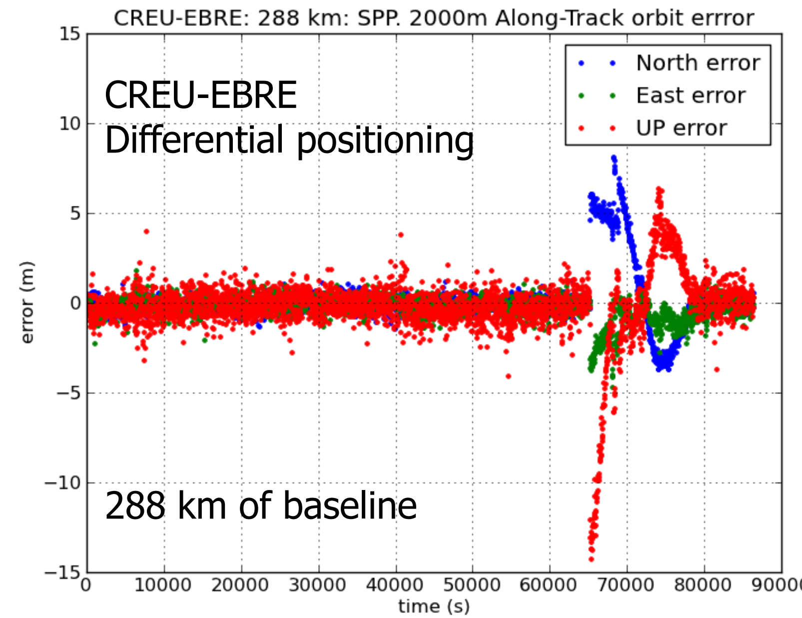
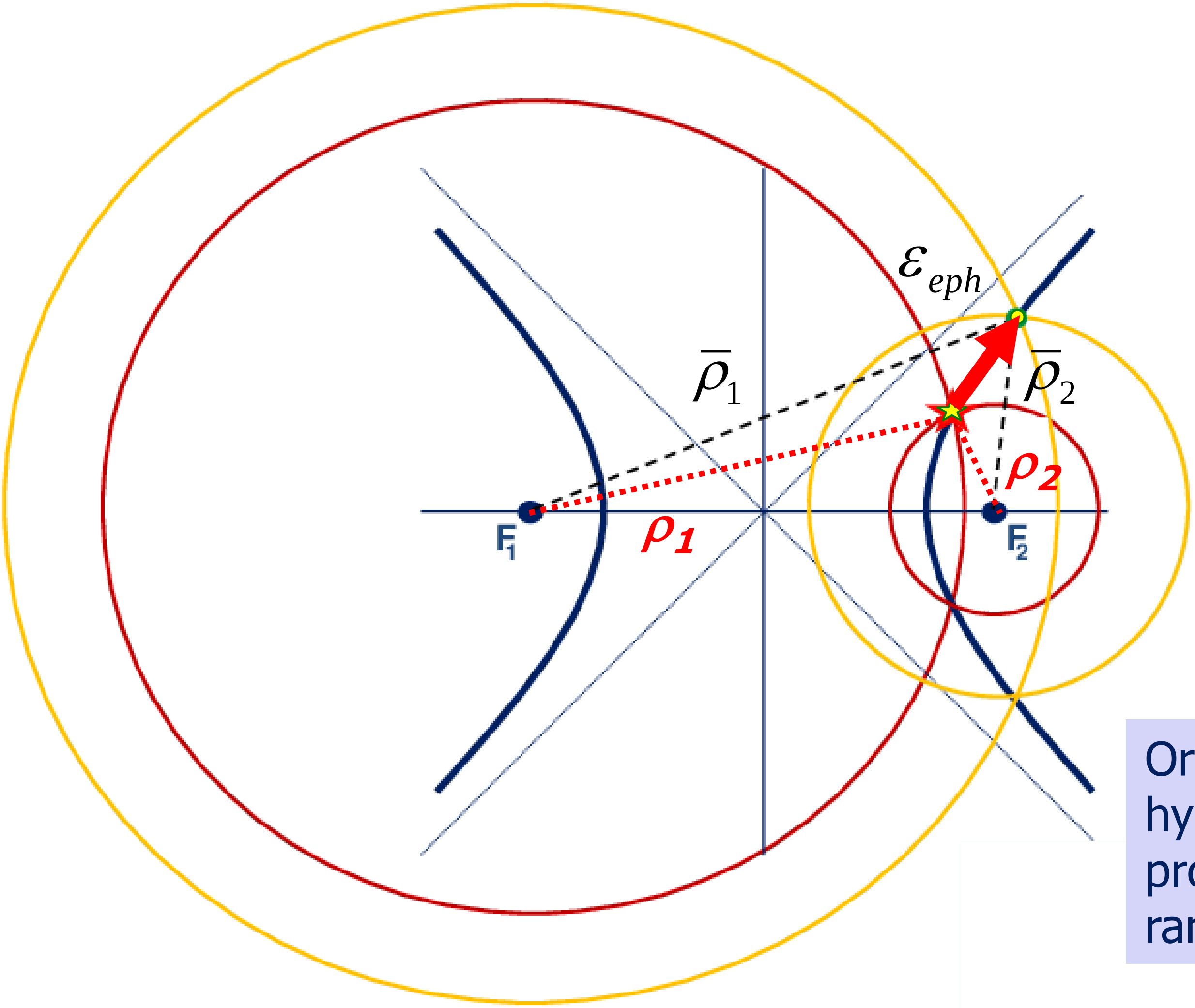




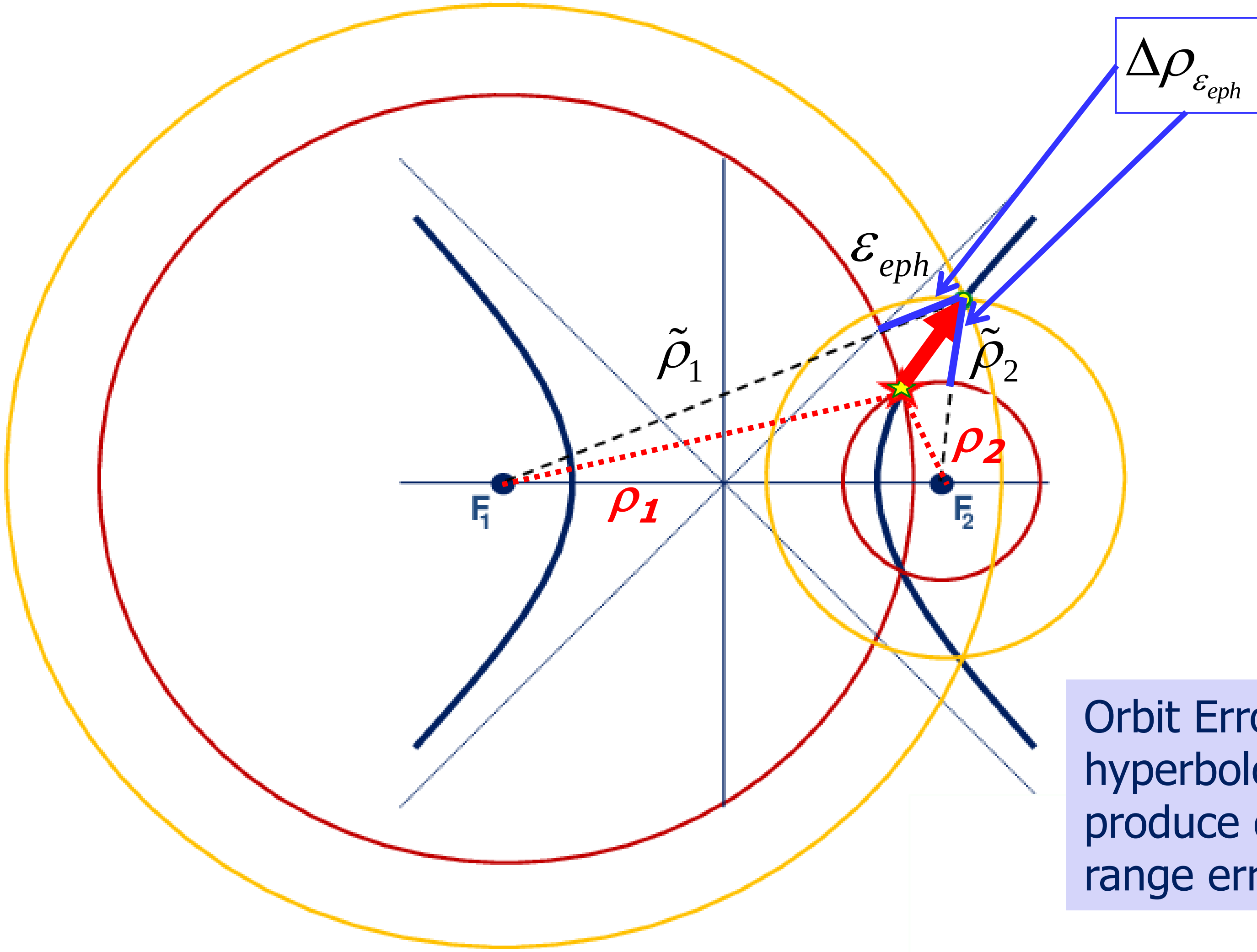






Orbit Errors over the hyperboloid will not produce differential range errors.

$$\bar{\rho}_1 = \rho_1 + \Delta\rho_{\varepsilon_{eph}}$$
$$\bar{\rho}_2 = \rho_2 + \Delta\rho_{\varepsilon_{eph}}$$
$$\Rightarrow \quad \bar{\rho}_1 - \bar{\rho}_2 = \rho_1 - \rho_2 = \text{constant} \Rightarrow \quad \delta\rho = 0$$



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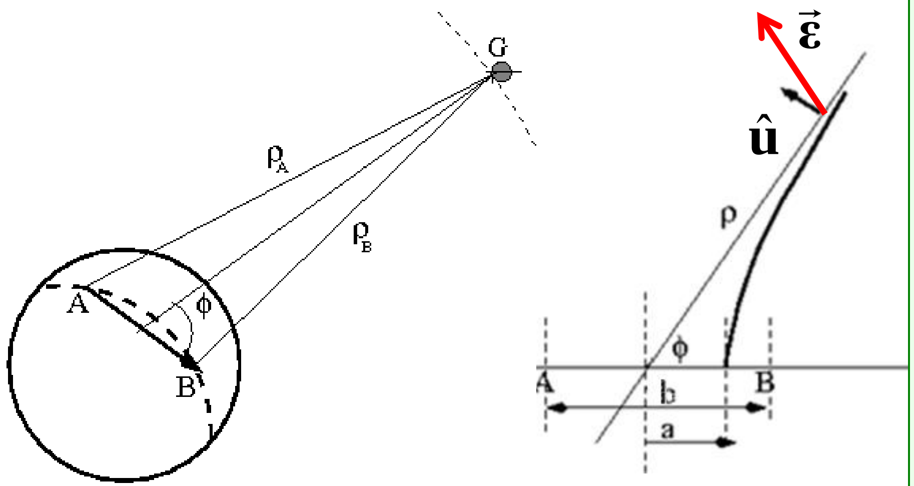


$a = (\rho_B - \rho_A)/2$: hyperboloid semiaxis

$b/2$: focal length

where $a = \frac{1}{2} b \cos\phi$

Note: in this 3D problem ϕ is NOT the elevation of ray.

- Errors over the hyperboloid (i.e. $\rho_B - \rho_A = ctt$) will not produce differential range errors.
- The highest error is given by the vector $\hat{\mathbf{u}}$, orthogonal to the hyperboloid and over the plain containing the baseline vector $\hat{\mathbf{b}}$ and the LoS vector $\hat{\boldsymbol{\rho}}$.

Note:
Being the baseline b much smaller than the distance to the satellite, we can assume that the LoS vectors from A and B receives are essentially identical to ρ.
That is, $\rho_B \cong \rho_A \cong \rho$

$$\mathbf{u} = \hat{\boldsymbol{\rho}} \times \left(\hat{\mathbf{b}} \times \hat{\boldsymbol{\rho}}\right) = \hat{\mathbf{b}}\left(\hat{\boldsymbol{\rho}}^T \cdot \hat{\boldsymbol{\rho}}\right) - \hat{\boldsymbol{\rho}}\left(\hat{\boldsymbol{\rho}}^T \cdot \hat{\mathbf{b}}\right)$$

$$= \mathbf{I}\,\hat{\mathbf{b}} - \left(\hat{\boldsymbol{\rho}} \cdot \hat{\boldsymbol{\rho}}^T\right)\hat{\mathbf{b}} = \left(\mathbf{I} - \hat{\boldsymbol{\rho}} \cdot \hat{\boldsymbol{\rho}}^T\right)\hat{\mathbf{b}}$$

Note: $\mathbf{u} = \sin\phi\, \hat{\mathbf{u}}$

Differential range error $\delta\rho$ produced by an orbit error $\varepsilon_{\parallel}$ parallel to vector $\hat{\mathbf{u}}$

Let $\delta\varepsilon \equiv \varepsilon_{\parallel}$

$$\delta\rho \equiv \delta(\rho_B - \rho_A) = 2\delta a =$$

$$= 2\frac{\partial a}{\partial \varepsilon}\delta\varepsilon = 2\frac{\partial a}{\partial \phi}\frac{\partial \phi}{\partial \varepsilon}\delta\varepsilon = -b\sin\phi\frac{\partial \phi}{\partial \varepsilon}\delta\varepsilon$$

$$\approx -b\sin\phi\frac{1}{\rho}\delta\varepsilon$$

Note: $\boldsymbol{\varepsilon}_{\parallel} \perp \boldsymbol{\rho} \Rightarrow \delta\varepsilon \approx \rho\,\delta\phi$

Note: being $\hat{\mathbf{u}}$ a vector orthogonal to the LoS $\hat{\boldsymbol{\rho}}$, thence, $\varepsilon_{\parallel} = \boldsymbol{\varepsilon}^T \hat{\mathbf{u}}$

Thence:

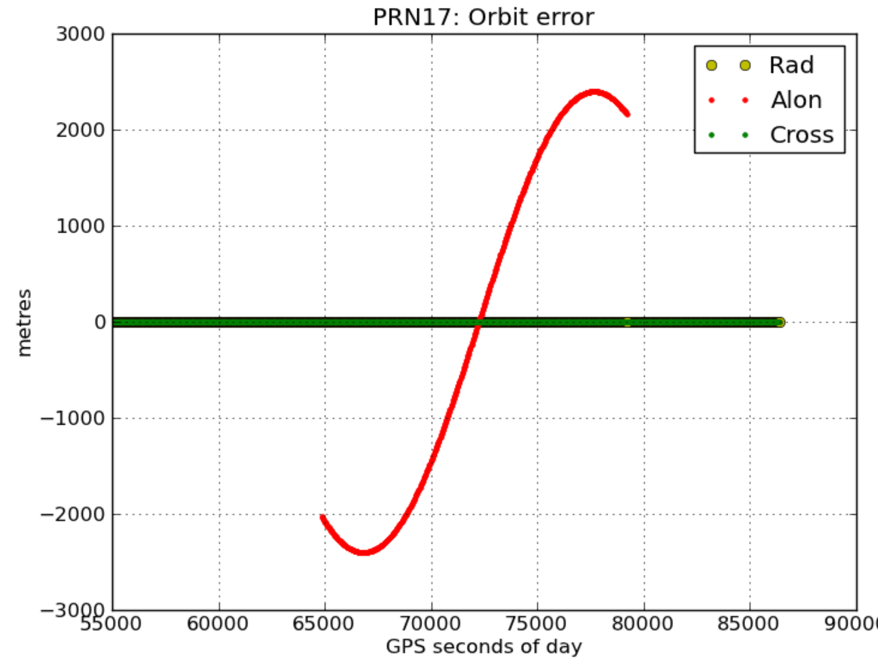
$$\delta\rho = -\frac{b\sin\phi}{\rho}\boldsymbol{\varepsilon}^T \cdot \hat{\mathbf{u}} = -\boldsymbol{\varepsilon}^T \cdot \left(\sin\phi\,\hat{\mathbf{u}}\right)\frac{b}{\rho}$$

$$= -\boldsymbol{\varepsilon}^T\left(\mathbf{I} - \hat{\boldsymbol{\rho}} \cdot \hat{\boldsymbol{\rho}}^T\right)\frac{\mathbf{b}}{\rho}$$

Where: $\mathbf{b} = b\,\hat{\mathbf{b}}$ is the baseline vector

ORBIT TEST : Broadcast orbits Along-track Error (PRN17)

PRN17:
Doy=077, Transm. time: 64818 sec

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    4.104180000000E+05 4.000000000000E+00


diff EPH.dat.org EPHcuc_x0.dat ----------------------------------------------
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----------------------------------------------------------------------------
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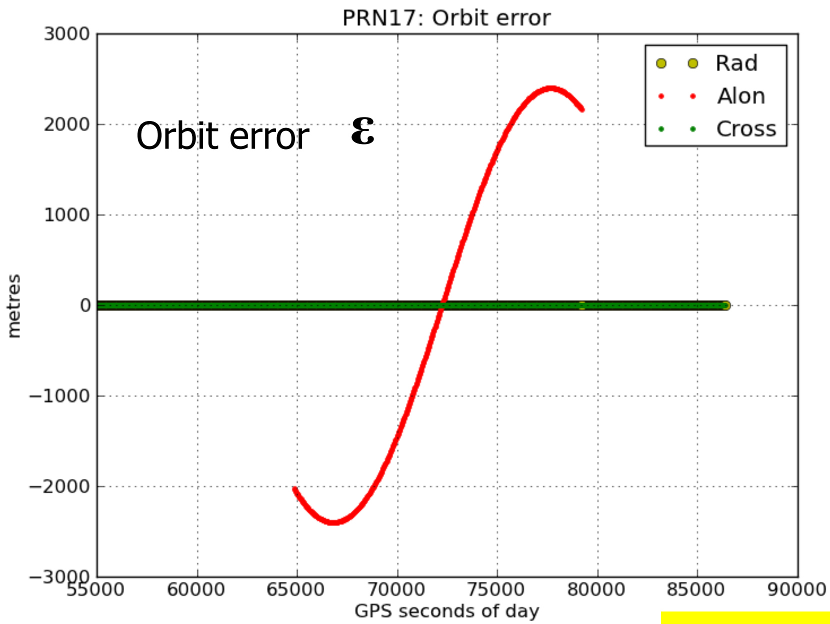
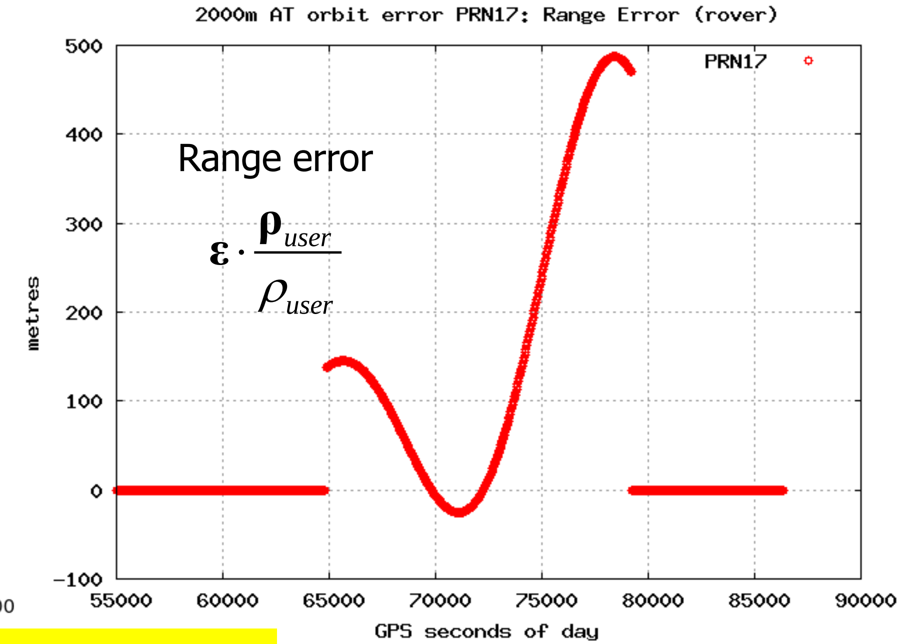
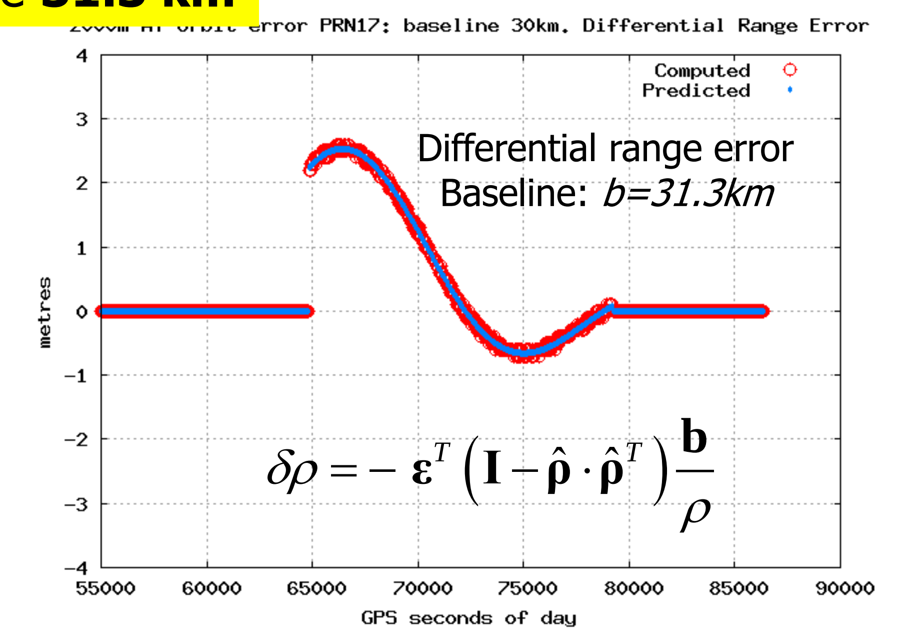




Baseline **31.3 km**



Contents

Error mitigation and short baseline concept

If the distance between the user and the reference station is "short enough", so that the residual error ionospheric, tropospheric and ephemeris are small compared to the typical errors due to receiver noise and multipath, it can be assumed:

$$T_{ru}^{j} = I_{ru}^{j} = 0\,; \qquad \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j} = 0$$

Note that the previous definition of "shortness" is quite fussy

Working with smoothed code, a residual error of about 0.5 metres could be tolerable, but for carrier based positioning it should be less than 1 cm to allow the carrier ambiguity fixing.

- The differential ephemeris error is at the level of few centimetres for baselines up to 100 km (i.e. 5 cm assuming a large bound of $\varepsilon_{eph}^{j} \approx 10\,\mathrm{m}$).
- The typical spatial gradient of the ionosphere (STEC) is 1-2 mm/km (i.e. 0.1-0.2 m in 100km), but it can be more than one order of magnitude higher when the ionosphere is active.

Error mitigation and short baseline concept

Note that the previous definition of "shortness" is quite fussy

- The correlation radio of the troposphere is lower than for the ionosphere. At 10km of separation the residual error can be up to 0.1-0.2 m. Nevertheless, 90% of the tropospheric delay can be modelled and the remaining 10% can be estimated together with the coordinates (for high precision applications). For distances beyond a ten of kilometres or significant altitude difference it would be preferable to correct for the tropospheric delay at the reference station and user receiver.

Carrier-smoothed code:
Pseudorange code measurement errors due to receiver noise and multipath can be reduced smoothing the code with carrier measurements.
Smoothed codes of 0.5m (RMS) can be obtained with 100 seconds smoothing.
On the other hand, the ionospheric error is substantially eliminated in differential mode and the filter can be allowed for lager time smoothing windows.

Contents

Differential code based positioning

If the reference station coordinates are known at the centimetre level and the distance between reference station and user are "not too large", we can assume

$$T_{ru}^{j} \approx 0 \, ; I_{ru}^{j} \approx 0$$

$$\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j} \approx 0$$

$$\boldsymbol{\varepsilon}_{site} \approx 0 \Rightarrow \Delta\mathbf{r}_{ru} \approx \Delta\mathbf{r}_{u}$$

Thence,

$$P_{ru}^{j} = \rho_{ru}^{j} + c\,\delta t_{ru} + T_{ru}^{j} + I_{ru}^{j} + K_{ru} + \nu_{P\,ru}^{\;j} \rightarrow P_{ru}^{j} = \rho_{ru}^{j} + c\,\delta t_{ru} + K_{ru} + \nu_{P\,ru}^{\;j}$$

$$\rho_{ru}^{j} = \rho_{0ru}^{j} - \hat{\boldsymbol{\rho}}_{0u}^{j} \cdot \Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{site,r} + \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j} \rightarrow \rho_{ru}^{j} = \rho_{0\,ru}^{\;j} - \hat{\boldsymbol{\rho}}_{0\,u}^{\;j} \cdot \Delta\mathbf{r}_{u}$$

(The terms T_{ru}^{j}, I_{ru}^{j}, $\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{site,r}$ and $\hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j}$ are crossed out.)

Note : for baselines up to 100 km *the range error of broadcast orbits is less than* 10 cm *(assuming* $\varepsilon_{eph}^{j} \approx 10\,\mathrm{m}$ *).*

Or, what is the same:

$$P_{ru}^{j} - \rho_{0\,ru}^{\;j} = -\hat{\boldsymbol{\rho}}_{0\,u}^{\;j} \cdot \Delta\mathbf{r}_{u} + c\,\delta t_{ru} + K_{ru} + \nu_{P\,ru}^{\;j}$$

The left hand side of previous equation can be spitted in two terms: one associated to the reference station and the other to the user:

$$P_{ru}^{j} - \rho_{0\,ru}^{\;j} = P_{u}^{j} - \rho_{0\,u}^{\;j} - \left(P_{r}^{j} - \rho_{0\,r}^{\;j}\right)$$

$$\Delta\mathbf{r}_{ru} = \Delta\mathbf{r}_{u} - \Delta\mathbf{r}_{r} = \Delta\mathbf{r}_{u} - \boldsymbol{\varepsilon}_{site}$$

Differential code based positioning

$$P_{ru}^{j} - \rho_{0\,ru}^{\;j} = -\hat{\boldsymbol{\rho}}_{0\,u}^{\;j} \cdot \Delta\mathbf{r}_u + c\,\delta t_{ru} + K_{ru} + \nu_{P\,ru}^{\;j}$$

The left hand side of previous equation can be spitted in two terms: one associated to the reference station and the other to the user:

$$P_{ru}^{j} - \rho_{0\,ru}^{\;j} = P_{u}^{j} - \rho_{0\,u}^{\;j} - \left(P_{r}^{j} - \rho_{0\,r}^{\;j}\right)$$

- The term $P_{r}^{j} - \rho_{0\,r}^{\;j}$ is the error in range measured by the reference station, which can be broadcasted to the user as a differential correction:

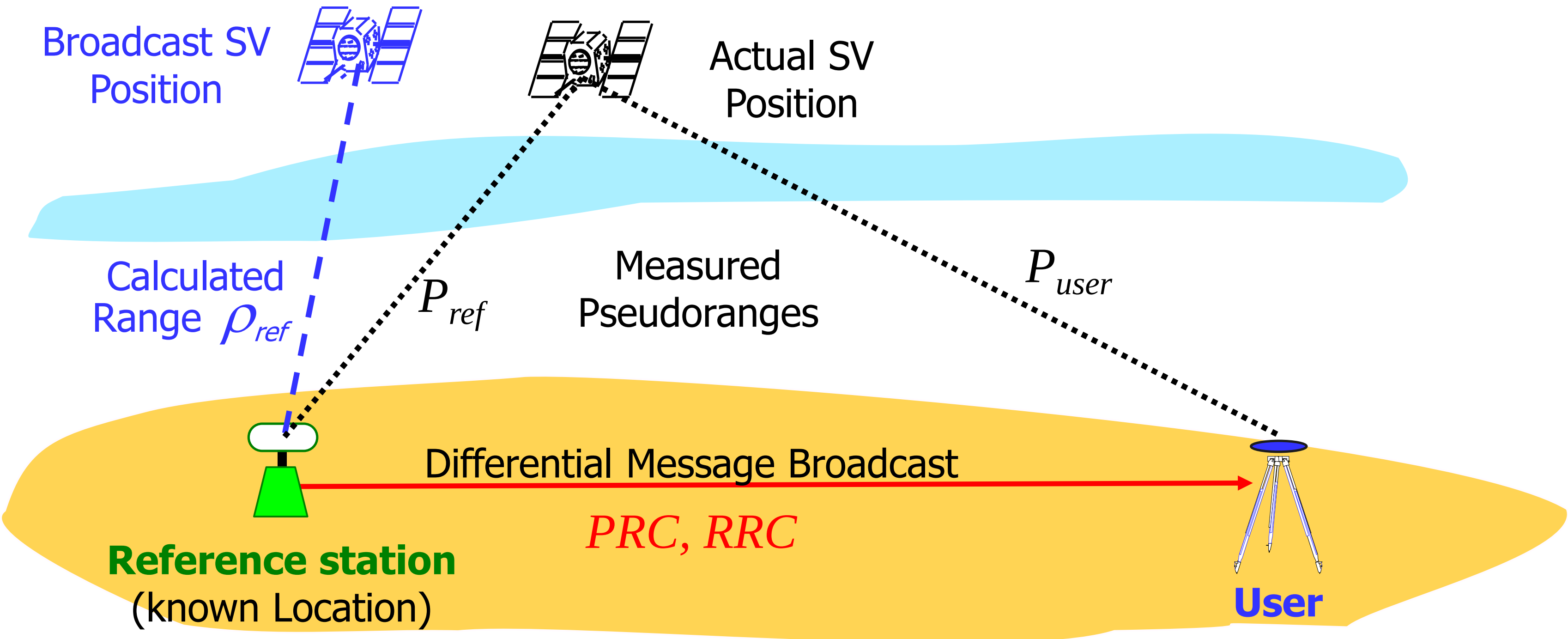
$$PRC^{j} = \rho_{0\,r}^{\;j} - P_{r}^{j}$$

- The user applies this differential correction to remove/mitigate common errors: $$P_{u}^{j} - \rho_{0\,u}^{\;j} + PRC^{j} = -\hat{\boldsymbol{\rho}}_{0\,u}^{\;j} \cdot \Delta\mathbf{r}_u + c\,\delta t_{ru} + \nu_{P\,ru}^{\;j}$$

Where the receiver's instrumental delay term K_{ru} is included in the differential clock $c\,\delta t_{ru}$

For distances beyond a ten of kilometres, or significant altitude difference, it would be preferable to correct for the tropospheric delay at the reference station and user receiver.

Range Differential Correction Calculation



- The **reference station** with known coordinates, computes pseudorange and range-rate corrections: $PRC = \rho_{ref} - P_{ref}$, $RRC = \Delta PRC / \Delta t$.

- The **user** receiver applies the PRC and RRC to correct its own measurements, $P_{user} + (PRC + RRC\,(t - t_0))$, removing SIS errors and improving the positioning accuracy.

DGNSS with code ranges: users within a hundred of kilometres can obtain **one-meter-level** positioning accuracy using such pseudorange corrections.

Differential code based positioning

The user applies this differential correction to remove/mitigate common errors:

$$P_u^j - \rho_{0\,u}^{\;j} + PRC^j = -\hat{\boldsymbol{\rho}}_{0\,u}^{\;j} \cdot \Delta \mathbf{r}_u + c\,\delta t_{ru} + \nu_{P\,ru}^{\;j}$$

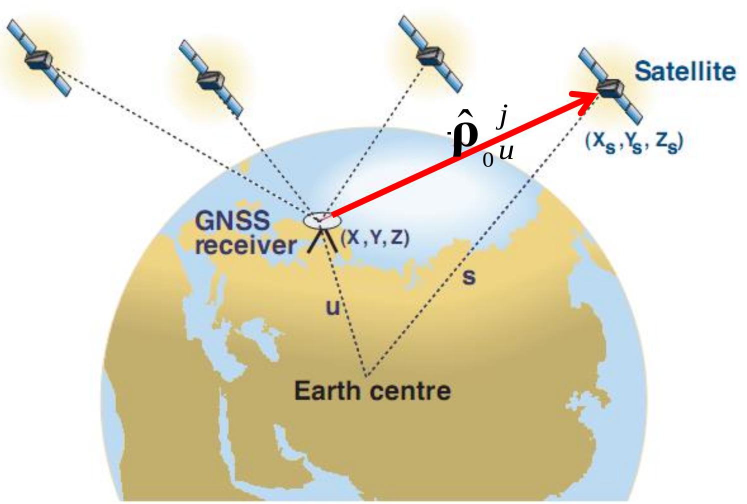
where the receiver's instrumental delay term K_{ru} is included in the differential clock $c\,\delta t_{ru}$

The previous system for navigation equations is written in matrix notation as:

$$\begin{bmatrix} \mathrm{Pref}^1 \\ \mathrm{Pref}^2 \\ \cdots \\ \mathrm{Pref}^n \end{bmatrix} = \begin{bmatrix} -\left(\hat{\boldsymbol{\rho}}_{0\,u}^{\;1}\right)^T & 1 \\ -\left(\hat{\boldsymbol{\rho}}_{0\,u}^{\;2}\right)^T & 1 \\ \cdots & \\ -\left(\hat{\boldsymbol{\rho}}_{0\,u}^{\;n}\right)^T & 1 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{r}_u \\ c\,\delta t_{ru} \end{bmatrix}$$

where

$$\mathrm{Pref}^j \equiv P_u^j - \rho_{0\,u}^{\;j} + PRC^j$$



Differential code based positioning

Time synchronization issues:

For simplicity we have dropped any reference to measurement epochs, but real-time implementations entail delays in data transmission and the time update interval can be limited by bandwidth restrictions.

- Differential corrections vary slowly and its useful life can be up to several minutes with S/A=off.
- To reduce bandwidth, the reference station computes Pseudorange Corrections (PRC) and Range-Rate Correction (RRC) for each satellite in view, which are broadcast to every several seconds, up to a minute interval with S/A=off.
- The user computes the PRC at the measurement epoch as:

$$PRC^{j}(t) = PRC^{j}(t_0) + RRC^{j}(t - t_0)$$

Differential code based positioning

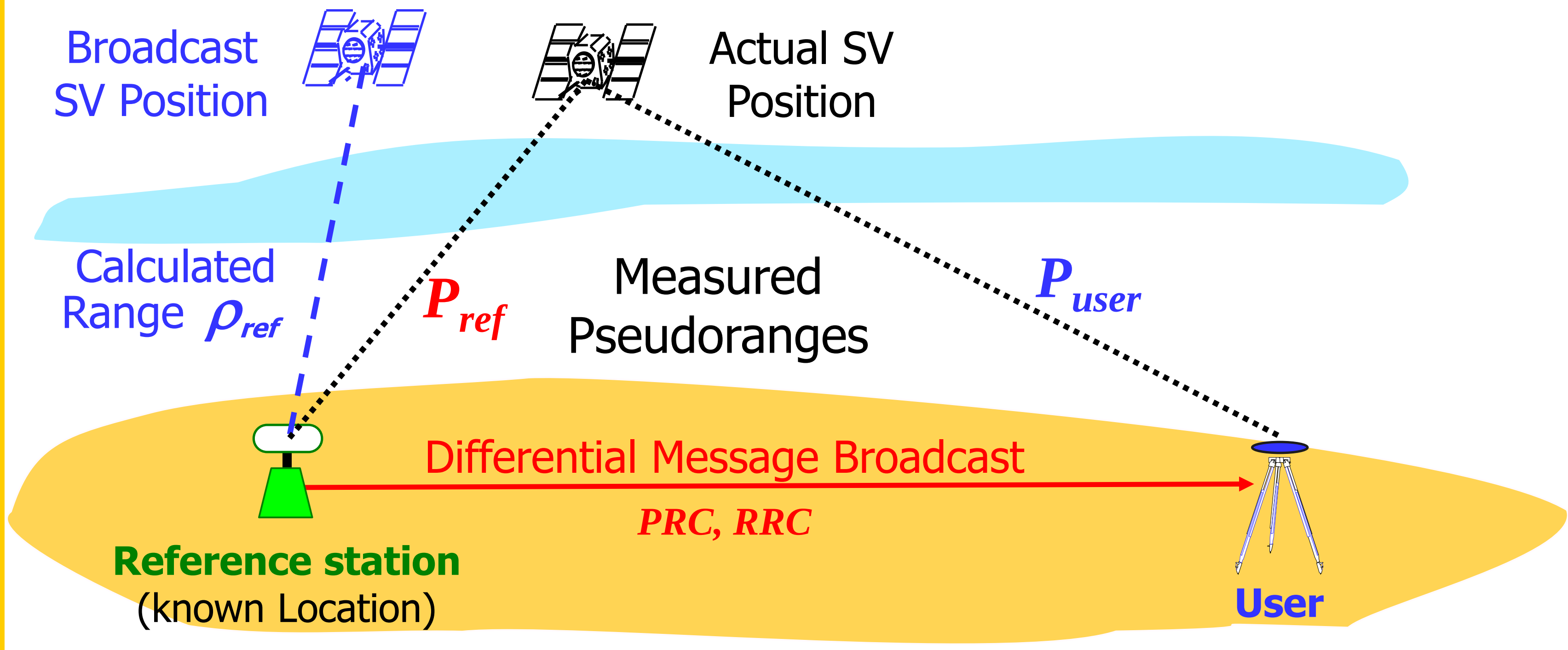
Data handling:

Reference station and user have to coordinate how the measurements are to be processed:

- Corrections must be identified with an Issue of Data (IOD) and time-out must be considered.
- Both receivers must use the same ephemeris orbits (which are identified by the IODE).
- If reference station uses a tropospheric model the same model must be applied by the user.
- If reference station uses the broadcast ionospheric model, the user must do the same.

Note: we have considered here only code measurements. The carrier based positioning will be treated next, using double differences of measurements and targeting the ambiguity fixing.

Range Differential Correction Calculation



– The reference station with known coordinates, computes pseudorange and range-rate corrections: $PRC = \rho_{ref} - P_{ref}$, $RRC = \Delta PRC / \Delta t$.

– The user receiver applies the PRC and RRC to correct its own measurements, $P_{user} + (PRC(t_0) + RRC(t-t_0))$, removing SIS errors and improving the positioning accuracy.

DGNSS with code ranges : users within a hundred of kilometres can obtain one-meter-level positioning accuracy using such pseudorange corrections.

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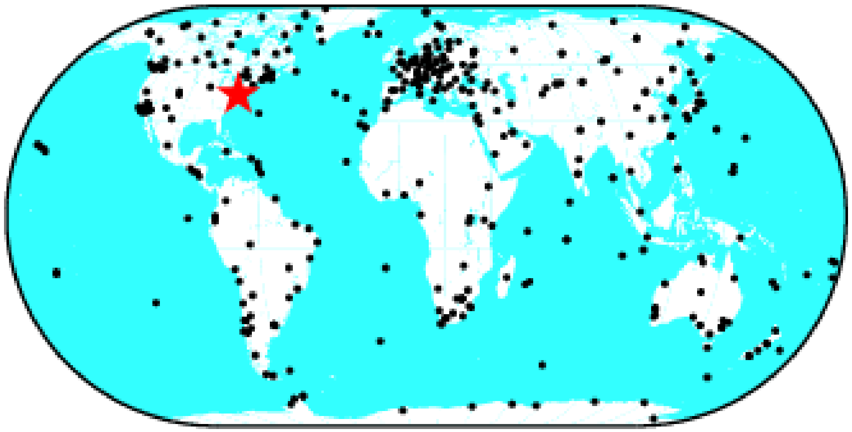

ftp://cddis.gsfc.nasa.gov/highrate/2013/

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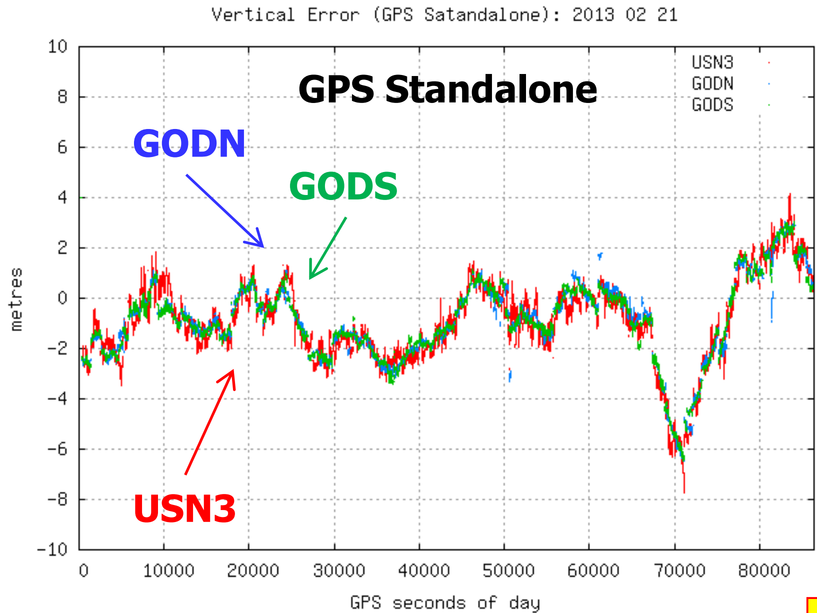
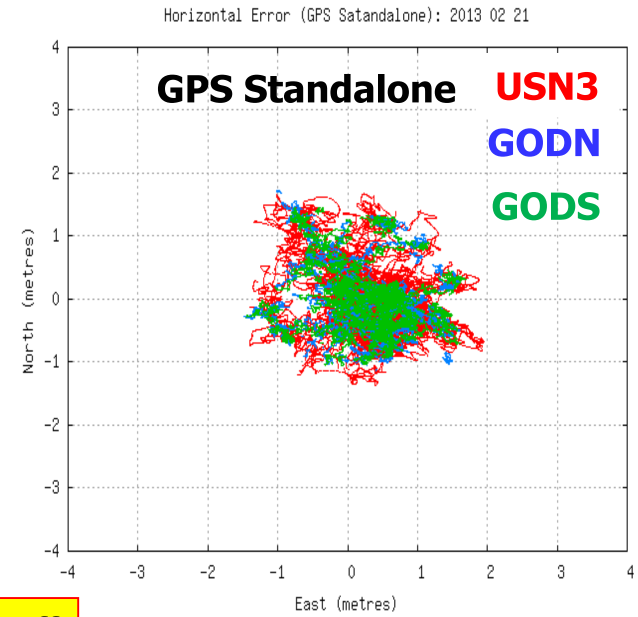
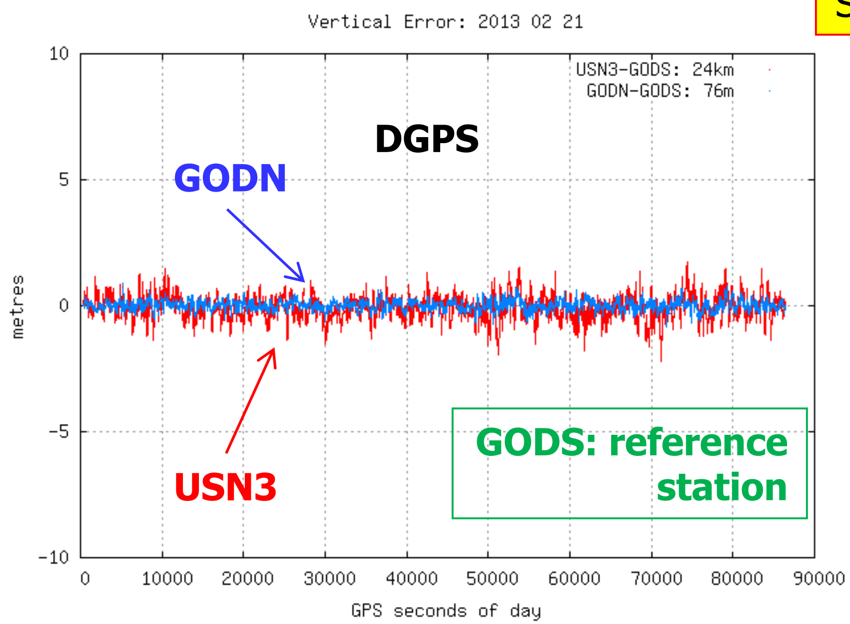
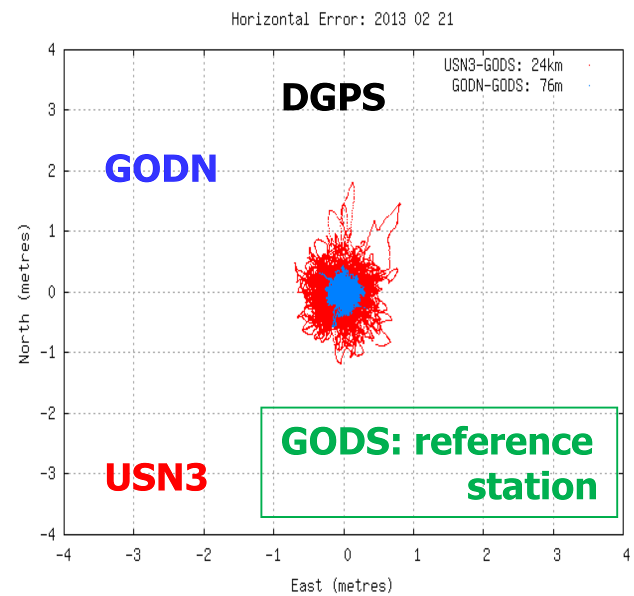

Vertical Error (GPS Satandalone): 2013 02 21
GPS Standalone
USN3
GODN
GODS
GODN
GODS
USN3
metres
GPS seconds of day


Horizontal Error (GPS Satandalone): 2013 02 21
GPS Standalone
USN3
GODN
GODS
North (metres)
East (metres)

S/A=off


Vertical Error: 2013 02 21
USN3-GODS: 24km
GODN-GODS: 76m
DGPS
GODN
USN3
GODS: reference station
metres
GPS seconds of day


Horizontal Error: 2013 02 21
USN3-GODS: 24km
GODN-GODS: 76m
DGPS
GODN
USN3
GODS: reference station
North (metres)
East (metres)

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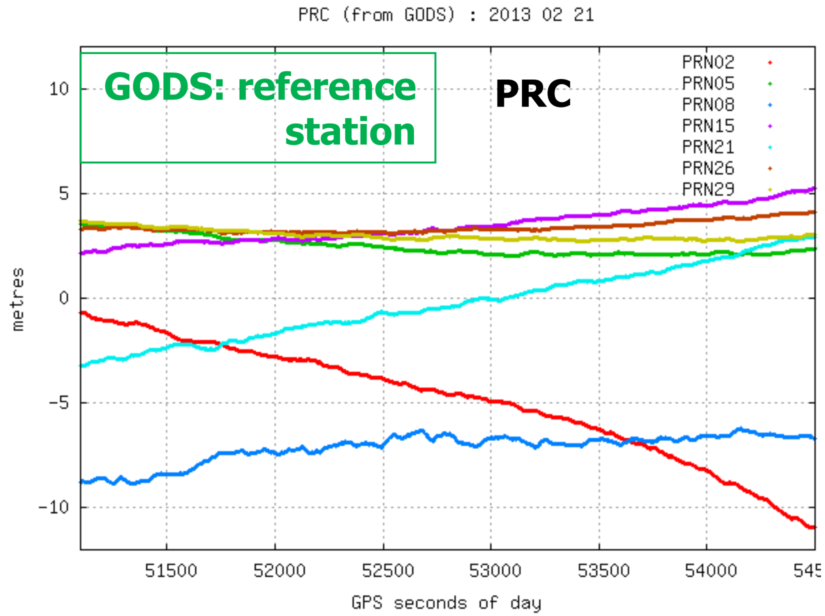

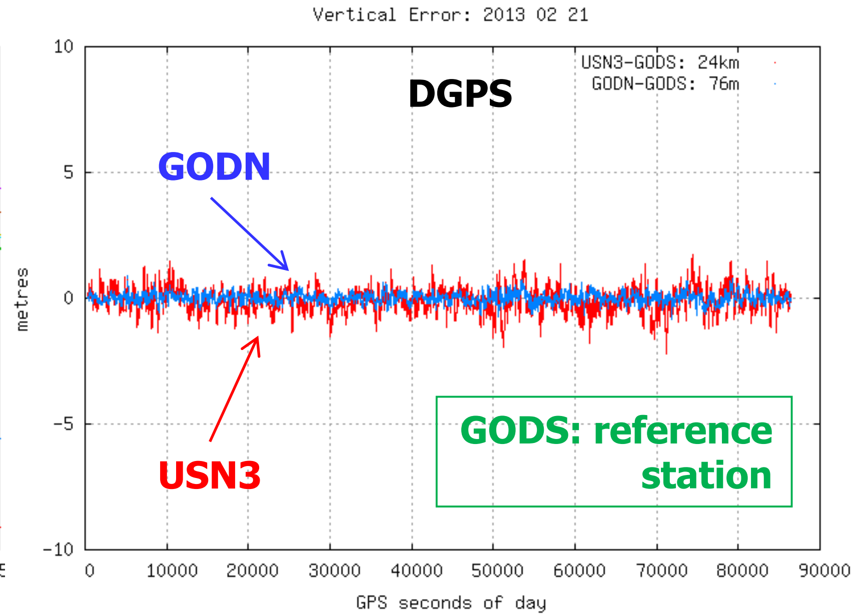

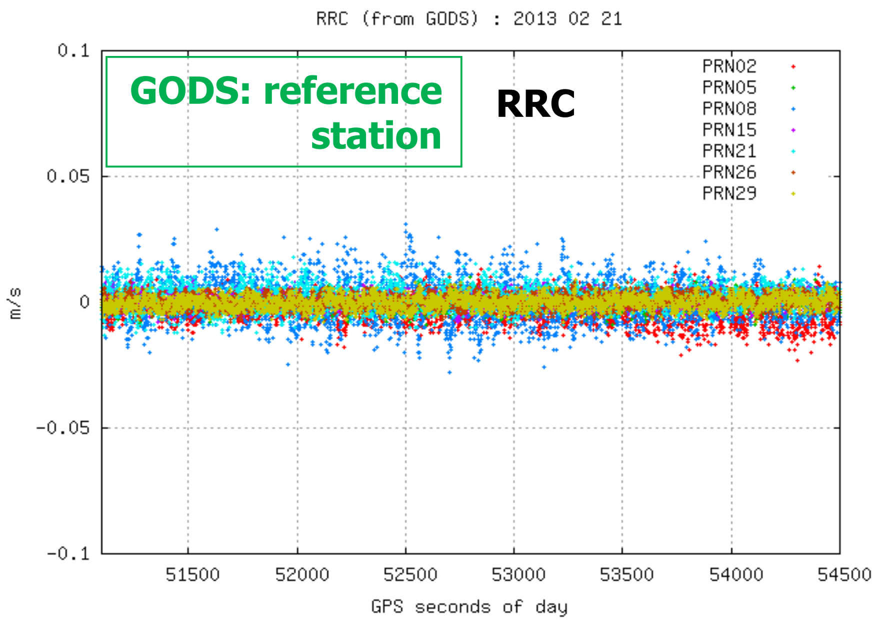

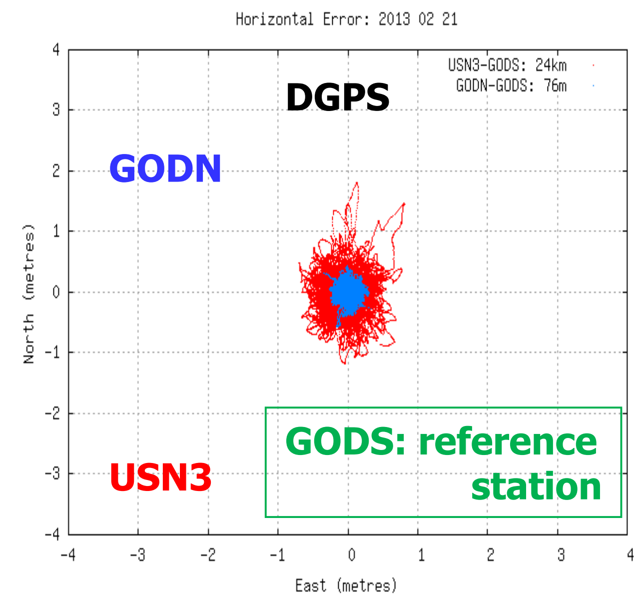

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[RD-1] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 1: Fundamentals and Algorithms. ESA TM-23/1. ESA Communications, 2013.

[RD-2] J. Sanz Subirana, J.M. Juan Zornoza, M. Hernández-Pajares, GNSS Data processing. Volume 2: Laboratory Exercises. ESA TM-23/2. ESA Communications, 2013.

[RD-3] Pratap Misra, Per Enge. Global Positioning System. Signals, Measurements, and Performance. Ganga-Jamuna Press, 2004.

[RD-4] B. Hofmann-Wellenhof et al. GPS, Theory and Practice. Springer-Verlag. Wien, New York, 1994.

[RD-5] Gang Xie, Optimal on-airport monitoring of the integrity of GPS-based landing systems, PhD Dissertation, 2004.

Thank you

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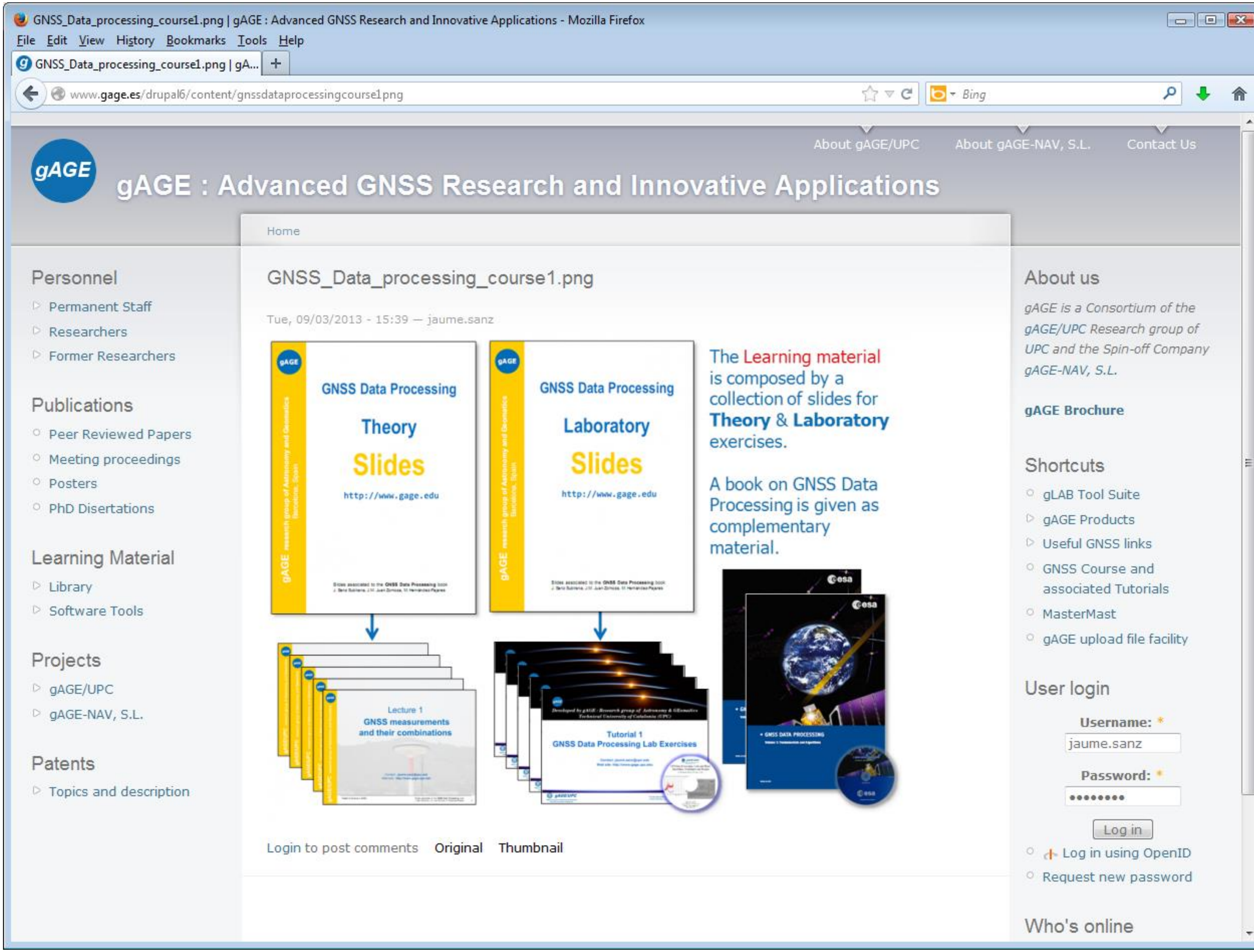

GNSS Data Processing, Vol. 1: Fundamentals and Algorithms.
GNSS Data Processing, Vol. 2: Laboratory exercises.

Backup Slides

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Linear model for Differential Positioning

Code and carrier measurements

$$P_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j + I_i^j + K_i + K^j + \nu_{P\,i}^{\;j}$$

$$L_i^j = \rho_i^j + c\left(\delta t_i - \delta t^j\right) + T_i^j - I_i^j + \lambda\,\omega_i^j + \lambda\,N_i^j + b_i + b^j + \nu_{L\,i}^{\;j}$$

where: $\rho_i^j = \rho_{0i}^j - \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}_i + \hat{\boldsymbol{\rho}}_{0i}^j \cdot \Delta\mathbf{r}^j$

Single difference

$$(\bullet)_{ru}^j \equiv \Delta(\bullet)_{ru}^j = (\bullet)_u^j - (\bullet)_r^j$$

$$P_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j + I_{ru}^j + K_{ru} + \varepsilon_{ru}^j$$

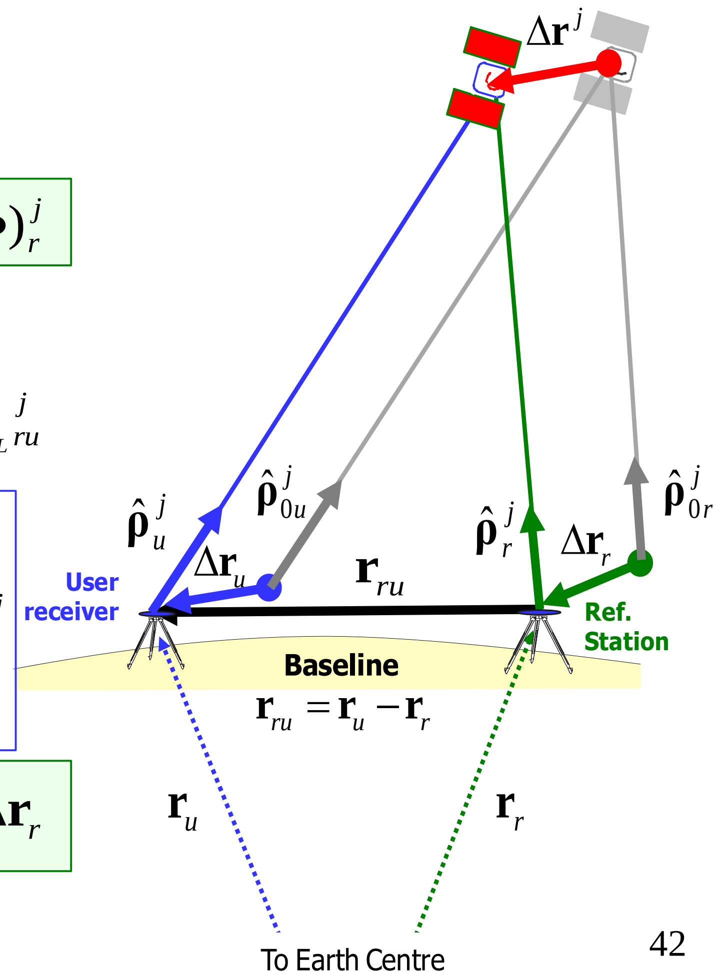
$$L_{ru}^j = \rho_{ru}^j + c\,\delta t_{ru} + T_{ru}^j - I_{ru}^j + \lambda\,\omega_{ru}^j + \lambda\,N_{ru}^j + b_{ru} + \nu_{L\,ru}^{\;j}$$

where: $\rho_{ru}^j = \rho_u^j - \rho_r^j$

$$\rho_{ru}^j = \rho_{0ru}^j - \hat{\boldsymbol{\rho}}_{0u}^j \cdot \Delta\mathbf{r}_u + \hat{\boldsymbol{\rho}}_{0r}^j \cdot \Delta\mathbf{r}_r + \hat{\boldsymbol{\rho}}_{0u}^j \cdot \Delta\mathbf{r}^j - \hat{\boldsymbol{\rho}}_{0r}^j \cdot \Delta\mathbf{r}^j$$

$$= \rho_{0ru}^j - \hat{\boldsymbol{\rho}}_{0u}^j \cdot \Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}_{0ru}^j \cdot \Delta\mathbf{r}_r + \hat{\boldsymbol{\rho}}_{0ru}^j \cdot \Delta\mathbf{r}^j$$

being: $\rho_{0ru}^j \equiv \rho_{0u}^j - \rho_{0r}^j$; $\Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_u - \Delta\mathbf{r}_r$



Exercise:

Let be: $\rho_{ru}^{j} = \rho_{u}^{j} - \rho_{r}^{j}$

where $\rho_{i}^{j} = \rho_{0i}^{j} - \hat{\boldsymbol{\rho}}_{0i}^{j} \cdot \Delta\mathbf{r}_{i} + \hat{\boldsymbol{\rho}}_{0i}^{j} \cdot \Delta\mathbf{r}^{j}$ (from Taylor expansion)

Show that the Single Differences are given by:

$$\rho_{ru}^{j} = \rho_{0ru}^{j} - \hat{\boldsymbol{\rho}}_{0u}^{j} \cdot \Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \Delta\mathbf{r}_{r} + \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \Delta\mathbf{r}^{j}$$

being: $\rho_{0ru}^{j} \equiv \rho_{0u}^{j} - \rho_{0r}^{j}$; $\Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_{u} - \Delta\mathbf{r}_{r}$

Linear model for Differential Positioning

$$\rho_{ru}^{j} = \rho_{0ru}^{j} - \hat{\boldsymbol{\rho}}_{0u}^{j} \cdot \Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \Delta\mathbf{r}_{r} + \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \Delta\mathbf{r}^{j}$$

with $\Delta\mathbf{r}_{ru} \equiv \Delta\mathbf{r}_{u} - \Delta\mathbf{r}_{r}$

Let's assume that:

- The satellite coordinates are known with an uncertainty $\Delta\mathbf{r}^{j} \equiv \boldsymbol{\varepsilon}_{eph}^{j}$
- The reference station coordinates are known with an uncertainty $\Delta\mathbf{r}_{r} \equiv \boldsymbol{\varepsilon}_{site}$ $(i.e.\ \mathbf{r}_{r} = \mathbf{r}_{0r} + \boldsymbol{\varepsilon}_{site})$

Thence, the user position can be computed from the $\Delta\mathbf{r}_{ru}$ estimate, with an error $\boldsymbol{\varepsilon}_{site}$, as:

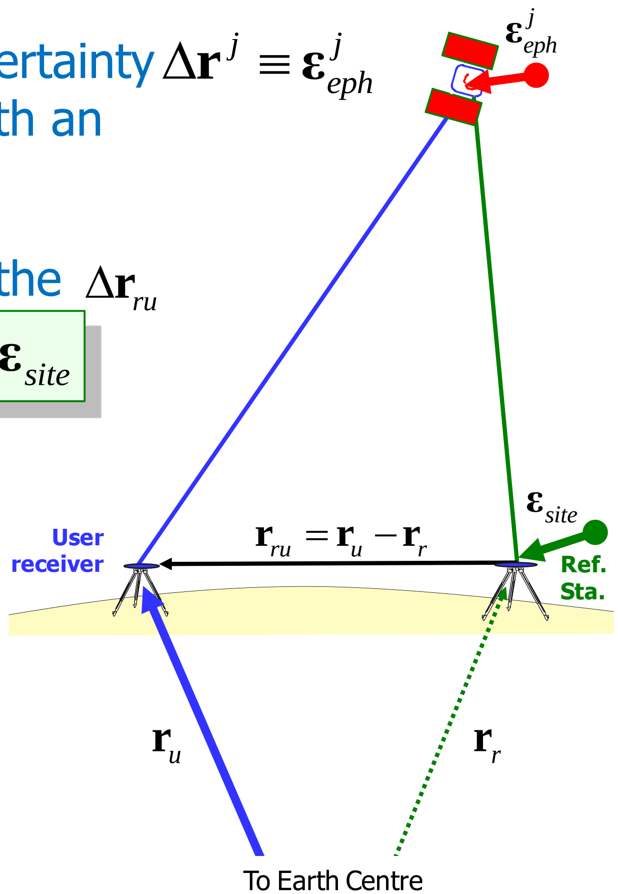
$$\mathbf{r}_{u} = \mathbf{r}_{0u} + \Delta\mathbf{r}_{ru} + \boldsymbol{\varepsilon}_{site}$$

and where the $\Delta\mathbf{r}_{ru}$ estimate will be affected in turn by the ephemeris and sitting site errors as:

$$\rho_{ru}^{j} = \rho_{0ru}^{j} - \hat{\boldsymbol{\rho}}_{0u}^{j} \cdot \Delta\mathbf{r}_{ru} - \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{site} + \hat{\boldsymbol{\rho}}_{0ru}^{j} \cdot \boldsymbol{\varepsilon}_{eph}^{j}$$

Range error due to **reference station** coordinates uncertainty

Range error due to **Sat. coordinates** uncertainty



$$\mathbf{r}_{u} = \mathbf{r}_{0u} + \Delta\mathbf{r}_{u} = \mathbf{r}_{0u} + \Delta\mathbf{r}_{ru} + \Delta\mathbf{r}_{r} = \mathbf{r}_{0u} + \Delta\mathbf{r}_{ru} + \boldsymbol{\varepsilon}_{site}$$

EGNOS Safety of Life Service Definition Document (Ref : EGN-SDD SoL, V1.0). European Comission.

http://www.essp-sas.eu/downloads/vubjj/egnos_sol_sdd_in_force.pdf

Error sources (1σ)	GPS - Error Size (m)	EGNOS - Error Size (m)
GPS SREW	4.0[12]	2.3
Ionosphere (UIVD error)	2.0 to 5.0[13]	0.5
Troposphere (vertical)	*0.1*	*0.1*
GPS Receiver noise	*0.5*	*0.5*
GPS Multipath (45º elevation)	*0.2*	*0.2*
GPS UERE 5º elevation	**7.4 to 15.6**	**4.2 (after EGNOS corrections)**
GPS UERE 90º elevation	**4.5 to 6.4**	**2.4 (after EGNOS corrections)**

[12] *GPS Standard Positioning Service Performance Standard [RD-3].*
[13] *This is the typical range of ionospheric residual errors after application of the baseline Klobuchar model broadcast by GPS for mid-latitude regions.*

SREW: Satellite Residual Error for the Worst user location.
UIVD: User Ionospheric Vertical Delay.
UERE: User Equivalent Range Error